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A New Analytical Approach to the Long Term Behaviour of Moisture Concentrations in Building Cavities—II Condensing Cavity

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A previous paper analysed a mathematical model of a non-condensing building cavity. This paper extends the analysis of the first paper to analyse the seasonal moisture behaviour of a condensing building cavity. Climate statistics are used to calculate the duration of the winter wet-up period, and a rate of condensation formula is integrated to give total winter condensation. Although engineering design calculations cannot yet be attempted, some illustrative examples are given based on field data. The results give preliminary verification of the model analysed in both papers.

NOMENCLATURE

A_i effective surface area for the flux of vapour to and from region i (m^2)
A_m effective surface area of hygroscopic material for the flux of vapour to and from the cavity (m^2)
$\bar{c}$ a weighted mean vapour concentration (kg m^{-3})
c_a yearly mean saturated cavity vapour concentration (kg m^{-3})
c_b maximum excursion of daily mean saturated cavity vapour concentration from c_a (kg m^{-3})
c_c maximum excursion of hourly mean saturated cavity vapour concentration from c_b (kg m^{-3})
c_f final vapour concentration in the cavity (kg m^{-3})
c_i vapour concentration in region i (kg m^{-3})
c_o vapour concentration in the cavity (kg m^{-3})
c_p specific heat at constant pressure of air (1.03×10^3 J kg^{-1} $°C^{-1}$ at 0°C)
c_{sat} saturated vapour concentration (kg m^{-3})
$\bar{c}_w$ winter mean cavity vapour concentration (kg m^{-3})
F_{io} air change rate from region i to the cavity (s^{-1})
F_{oi} air change rate from the cavity to region i (s^{-1})
$\mathscr{K}$ dimensionless form of k, a proportionality constant linearising the sorption curve of the hygroscopic material
m moisture concentration in the hygroscopic material (kg m^{-3})
m_f final moisture concentration of the hygroscopic material (kg m^{-3})
m'_f moisture concentration in the hygroscopic material at the end of the summer drying period (kg m^{-3})
M_o original hygroscopic material moisture concentration (kg m^{-3})
p number of seasons for cavity moisture concentrations to reach equilibrium.
p_i water vapour partial pressure in region i (N m^{-2})
p_{sat} saturated water vapour pressure in the cavity (N m^{-2})
r_i vapour resistance from the cavity to region i (N s kg^{-1})
R universal gas constant (8310 J K^{-1} $kmole^{-1}$)
R_i thermal resistance (°C m^2 W^{-1})
S moisture concentration absorbed by the hygroscopic material during winter wet-up period (kg m^{-3})
S' total moisture concentration accumulated in the cavity during the winter wet-up period (kg m^{-3})
t time(s)
t_d duration of the drying period(s)
t_{day} number of seconds in a day(s)
t_o time constant associated with cavity performance(s)
t_m time constant associated with hygroscopic material performance(s)
t_w duration of winter wet-up period(s)
t_y number of seconds in a year(s)
t_1, t_2 time constant(s)
T Kelvin temperature (K)
$\bar{T}$ mean time and space Kelvin temperature (K)
V_o volume of the cavity (m^3)
W molecular weight of water (18 kg $kmole^{-1}$)
γ cavity condensation rate (kg m^{-3} s^{-1})
θ temperature (°C)
θ_i temperature in region i (°C)
$\bar{\theta}_i$ effective temperature in region i for heat transfer purposes (°C)
θ_o temperature in the cavity (°C)
μ ratio of the surface area of the hygroscopic material to the surface area of the cavity
ν ratio of the volume of the hygroscopic material to the volume of the cavity
ρ_a density of air (1.293 kg m^{-3} at 0°C)
τ dimensionless time

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1. INTRODUCTION

AN ANALYTICAL model describing the moisture behaviour of a building cavity containing a hygroscopic storage material was developed in the first paper of this series [1].

The long term drying (or wetting) behaviour of this cavity was investigated under non-condensing conditions. However, in practice under more severe climatic driving forces considerable amounts of condensation may appear in the cavity during winter. Furthermore, conditions are often such that more moisture condenses into the cavity during winter than evaporates out of it, giving rise to a net accumulation of moisture into the cavity during this winter wet-up period. If conditions are this severe, the possibility exists of more moisture accumulating each winter than dries out in the following summer; leading to serious moisture problems, occupant discomfort, and possibly even irreversible structural damage.

This work is concerned with extending the analysis of the first paper to describe approximately the moisture performance of a condensing cavity. To do so, the effects of temperature variation must be studied, a topic which was deliberately ignored in the first paper. Keiper *et al.* [2] calculate winter moisture accumulation in the condensing situation by guessing an average winter length and associated mean climate conditions. This approach is subjective, requiring experience and judgement of the location in question. In this paper the moisture accumulation is calculated objectively by using the results of the first paper [1], and imposing upon it some structure based on field experience as described in Section 2. Climate statistics are used to define a suitable winter wet-up period, and the resulting moisture accumulation calculated.

The results derived, while approximate only for the condensing cavity, taken together with those of the first paper, provide the designer with formulae to calculate every important cavity moisture parameter, such as:

(i) time required to reach steady state (defined to mean a seasonal repetition of moisture contents);
(ii) summer minimum and winter maximum moisture contents;
(iii) winter wet-up time;
(iv) effect of changing leakage rates and building materials;
(v) effect of the hygroscopicity of the storage medium;
(vi) effect of the ratio of cavity volume to storage volume.

This paper concludes with several calculational examples to indicate the utility of the formulae derived. These examples relate to field data from moisture control remedial measures undertaken by the Building Research Association of New Zealand (BRANZ) [3], and give some preliminary verification of the model.

2. CONDENSING CAVITY FORMULAE DERIVATION

The effects of temperature and cavity condensation, deliberately ignored in the first paper, are of prime concern in assessing cavity moisture performance. However, since the processes involved here are complex, transitory and nonlinear, a direct analytical approach does not seem possible.

In order to maintain the aim set out in the first paper of achieving useful analytical results, a certain amount of structure is imposed upon the problem from field experience. Specifically, it is assumed that the cavity construction and driving climate cause the cavity to wet-up in the winter and dry out in the summer, giving rise to drying curves of the type illustrated in Figs 1 and 2. The seasonal fluctuations of vapour concentration are introduced at this point to give a definition of winter wet-up period. This is assumed the same length from year to year, and that as a consequence, the same amount of moisture is accumulated each winter. These assumptions are also implicit in the Keiper [2] and Glaser methods [4].

This is sufficient structure to allow the calculation of the total concentration of moisture, S, accumulated in the hygroscopic material each winter by integrating the net condensation rate through this time. This brings in the seasonal variation in vapour concentration and the corresponding climate statistics, which is more satisfactory than guessing the winter length, as in the case of the Keiper method [2]. The approach to equilibrium in the condensing case can now be calculated, allowing formulae to be derived giving time to equilibrium, equation (11), and the final equilibrium moisture concentrations (summer and winter), equations (6) and (7). An interesting, if not surprising, result that comes out of this analysis is that the time it takes for a cavity to reach equilibrium is inversely proportional to the summer drying duration.

Definition of winter wet-up period

The model of the building cavity is the same as that used in the previous paper [1], i.e. there are n regions of moisture sources/sinks with water vapour transport between region i and the cavity region o, being mediated by water vapour partial pressure difference p_i–p_o and by air convention at a rate F_{io} into the cavity from region i and F_{oi} out of the cavity into region i (measured in s^{-1}). The cavity has

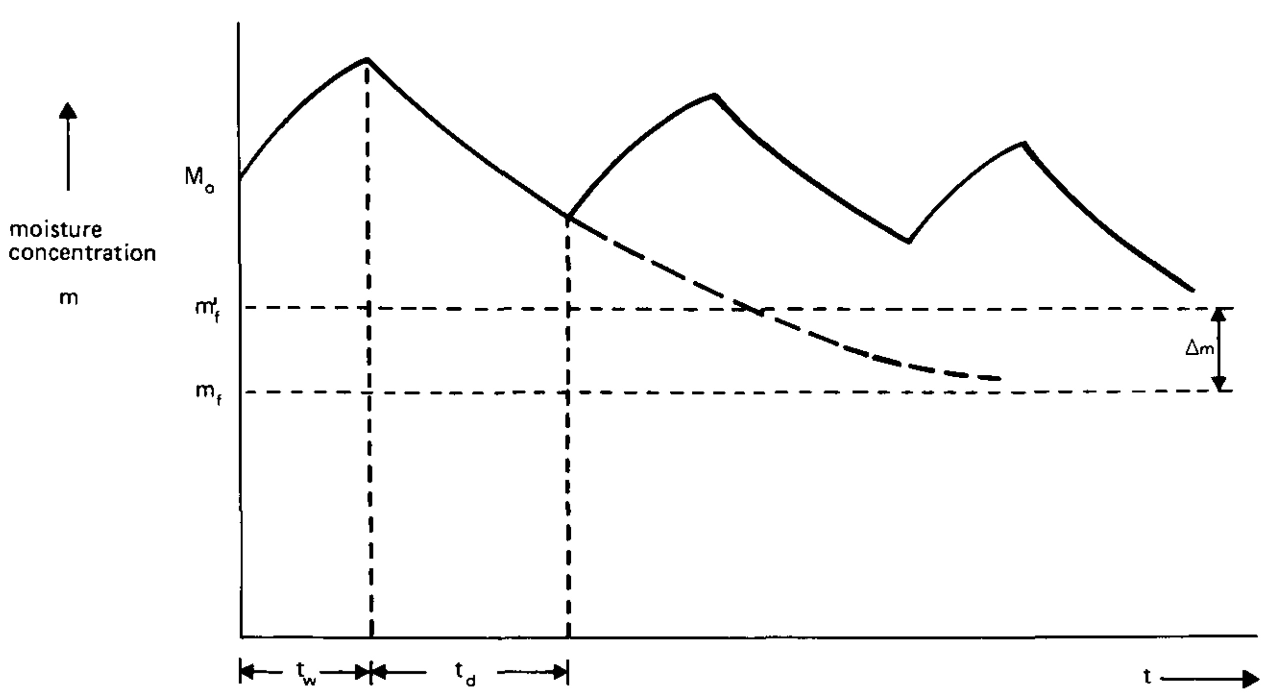

Fig. 1. Seasonal drying/wetting curves for a condensing cavity ($M_o > m_f'$).

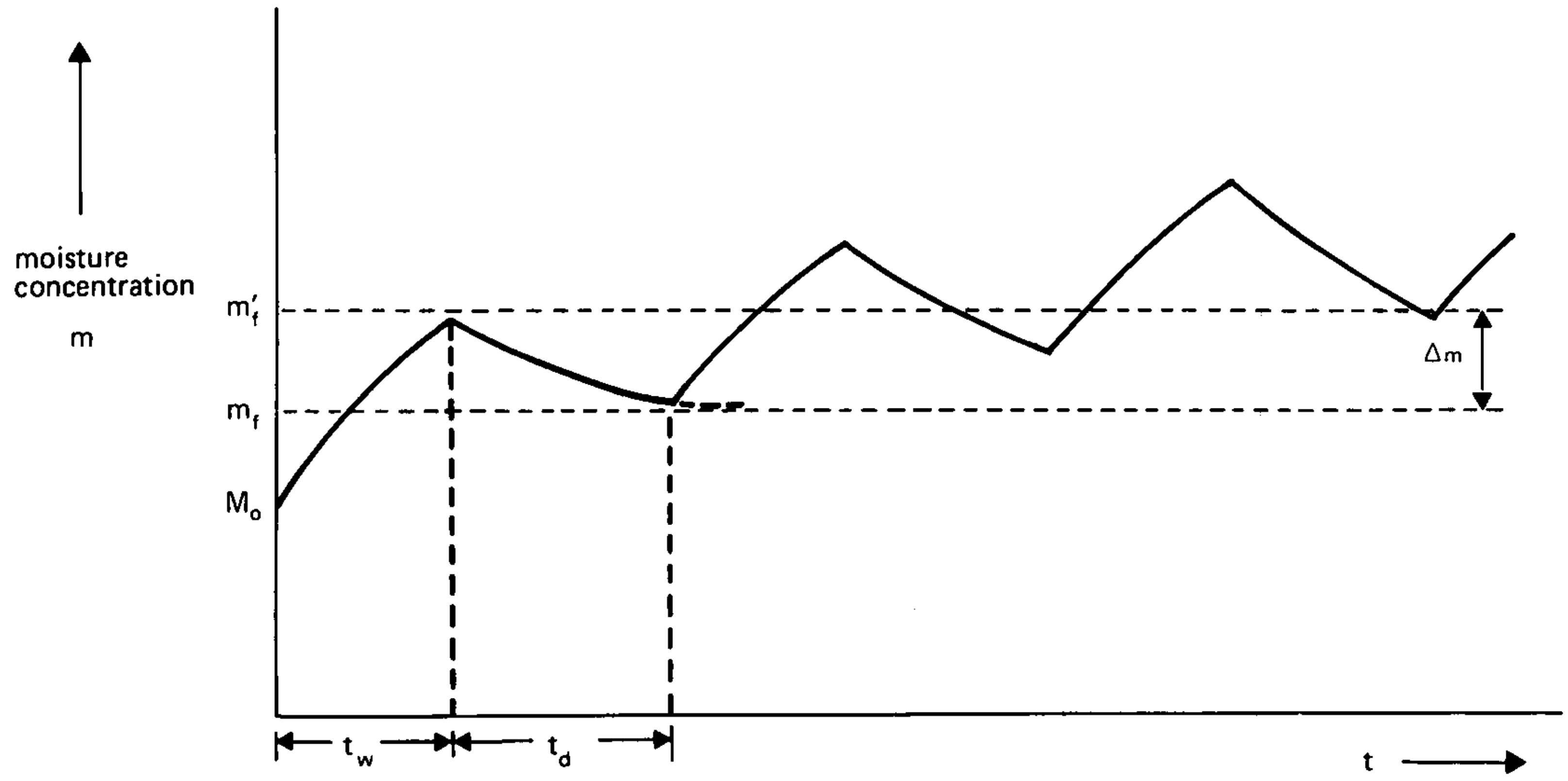

Fig. 2. Seasonal drying/wetting curves for a condensing cavity ($M_o < m'_f$).

volume V_o. As before, $\bar{T}$ is a mean temperature, time and region independent.

Cavity moisture concentration varies through time, as moisture enters and leaves the cavity and as the temperature varies. However, there will be some average cavity vapour concentration $\bar{c}_w$ throughout a given winter.

The external temperature variation in a given climate has two approximate sinusoidal components; one for the yearly temperature variation, and one for the daily temperature variation. So in turn does the cavity temperature, but with reduced amplitudes and some lag, as determined by the thermal properties of its linings, indoor temperature and heat transferred by leakage from other regions.

By conserving energy, the mean cavity temperature can be shown to be approximately

$$\theta_o = \frac{\sum_i \left(\frac{A_i \tilde{\theta}_i}{R_i} + \rho_a c_p V_o F_{io} \theta_i \right)}{\sum_i \left(\frac{A_i}{R_i} + \rho_a c_p V_o F_{oi} \right)}, \tag{1}$$

where θ_i is the air temperature in region i and $\tilde{\theta}_i$ is the equivalent temperature of region i for heat transfer purposes; for example, the external sol-air temperature.

Air leakage can be ignored for heat transfer purposes if

$$F_{io} \quad \text{and} \quad F_{oi} \ll \frac{A_i \tilde{\theta}_i}{V_o \rho_a c_p R_i \theta_i} \quad \text{for all } i.$$

This implies that for air leakage to be ignored for heat transfer purposes, it should be much less than 1 hr^{-1} for skillion roofs, or 0.1 hr^{-1} for pitched roofs.

Associated with this cavity temperature θ_o, there is a corresponding saturated vapour pressure and saturated vapour concentration. By linearising the temperature-saturated vapour concentration relationship in the temperature range of interest, it can be approximately said that

$$c_{sat} = c_a - c_b \cos \frac{2\pi t}{t_y} - c_c \cos \frac{2\pi t}{t_{day}}, \tag{2}$$

where c_{sat} = maximum possible vapour concentration at time t, c_a = yearly mean saturated vapour concentration in the cavity, $c_a - c_b$ = minimum daily mean saturated vapour concentration in the cavity, $c_a - c_b - c_c$ = minimum hourly mean saturated vapour concentration in the cavity, t_{day} = number of seconds in a day, t_y = number of seconds in a year.

By definition, condensing occurs whenever the saturated vapour concentration drops below the actual vapour concentration.

For the purposes of this work, the winter wet-up period is defined as that period of time (if it exists) when the daily average cavity saturated vapour concentration is below the winter mean cavity vapour concentration.

This requires $c_a - c_b \cos \frac{2\pi t}{t_y} \leqslant \bar{c}_w$,

i.e. $-\frac{1}{2}t_w \leqslant t \leqslant \frac{1}{2}t_w$,

where winter wet-up duration t_w is given by

$$t_w = \frac{t_y}{\pi} \cos^{-1}\left(\frac{c_a - \bar{c}_w}{c_b} \right). \tag{3}$$

In making this definition, it is appreciated that during spring and autumn the cavity will condense moisture for short intermittent periods, while during the winter the cavity will have short periods of evaporation. These finer details have been ignored on the assumption that their contribution to the net moisture accumulation is not large.

We can therefore assume, with Keiper [2], that the net amount of vapour entering the cavity during the winter wet-up period condenses. While it is accepted that this is only approximately true, it does allow both a well-defined winter wet-up period which can be used in any locality where the appropriate climate statistics are known, and at least a relative comparison to be made of the amount of moisture condensed in different locations. A more accurate assessment of winter moisture accumulation requires a numerical model whose predictions are backed up with field data.

Total winter condensation

By the definition given of the wet-up period, cavity relative humidity throughout this time is 100% and the net of all moisture entering the cavity must condense.

The condensation rate γ is given by

$$\gamma = \sum_i \left[\frac{A_i}{V_o}\left(\frac{p_i - p_{\text{sat}}}{r_i}\right) + F_{io}c_i - F_{oi}c_{\text{sat}}\right]$$
$$= \sum_i \left[\left(\frac{R\bar{T}}{V_o W}\frac{A_i(c_i - c_{\text{sat}})}{r_i}\right) + F_{io}c_i - F_{oi}c_{\text{sat}}\right]$$
$$= \frac{1}{t_o}(\bar{c} - c_{\text{sat}}),$$

where c_{sat} (p_{sat}) is the cavity saturated vapour concentration (pressure), and is a function of cavity temperature, see equation (2), t_o is the cavity response time constant defined by formula (7) in [1], and $\bar{c}$ is the mean cavity vapour concentration as given by formula (13) in [1].

The total condensation S' condensed (in kg m^{-3}) throughout the winter wet-up period must be

$$S' = \int_{-1/2t_w}^{1/2t_w} \gamma \, dt$$
$$= \frac{1}{t_o}\int_{-1/2t_w}^{1/2t_w}\left[\bar{c} - \left(c_a - c_b\cos\frac{2\pi t}{t_y} - c_c\cos\frac{2\pi t}{t_{\text{day}}}\right)\right] dt.$$

If $c_c t_{\text{day}} \ll c_b t_y$, which will be so, the diurnal integral can be ignored. Physically, we are saying that diurnal fluctuations integrate to zero over the winter wet-up period. Evaluating the other integrals in full, S' then comes to

$$S' = \frac{t_y}{\pi t_o}\left\{(\bar{c} - c_a)\cos^{-1}\left(\frac{c_a - \bar{c}_w}{c_b}\right) + c_b\sqrt{\left[1 - \left(\frac{c_a - \bar{c}_w}{c_b}\right)^2\right]}\right\}, \quad (4)$$

using the definitions of the various quantities involved.

Some of this condensation will condense on the hygroscopic material, to be absorbed there by the comparatively fast process of capillary action, the reverse of the process taking place when the material is drying under the constant rate regime. It is not clear what proportion of the cavity condensation enters the hygroscopic material, but the simplest plausible assumption is that it is in the ratio, μ, of the surface area of the hygroscopic material to the surface area of the cavity.

Under this assumption:

Amount of condensation entering the hygroscopic material

$$= \mu V_o S'.$$

Concentration of condensation in the hygroscopic material

$$= S = \frac{\mu V_o S'}{V_m} = \frac{\mu S'}{v}.$$

If the hygroscopic material is timber, then the ratio μ/v is 20–30 for large volume cavities with v of the order of 10^{-2}, and 2–3 for small volume cavities with v of the order of 10^{-1}. This is because the amount of framing timber remains approximately the same for both cavity sizes.

However, given the difficulty in predicting the proportion of condensation that finally appears in the hygroscopic material, and the difficulty in estimating the winter wet-up period, μ/v could be treated as an adjustable parameter, if necessary. A value of 15 seems appropriate for pitched roofs in New Zealand conditions.

Finally, then

$$S = \frac{\mu t_y}{v\pi t_o}\left\{(\bar{c} - c_a)\cos^{-1}\left(\frac{c_a - \bar{c}_w}{c_b}\right) + c_b\sqrt{\left[1 - \left(\frac{c_a - \bar{c}_w}{c_b}\right)^2\right]}\right\}. \quad (5)$$

Once steady state has been achieved, $\bar{c}_w$ could be taken as c_f.

Two observations are worth making at this point. Firstly, this function is sensitive to the values of the parameters chosen. This reflects the physical observation that two apparently similar cavities in similar environments can have quite different moisture performances. Secondly, it can be seen that S is inversely proportional to t_o; in other words, tight cavities accumulate less winter moisture. This is due to the simple fact that moisture enters the cavity more slowly. On the other hand, it is shown below that tight cavities have a higher end of summer moisture content. Overall then, tight cavities drying under a tighter regime may have slightly higher average moisture contents, but lower winter peaks, than cavities drying under the hygroscopically controlled regime.

Steady state moisture contents

Each winter the cavity receives an amount of condensation S' (kg m^{-3}), a proportion of which finds its way into the hygroscopic material; and then each summer undergoes a drying whose rate has been determined in the previous paper [1]. The process repeats itself, and it remains to calculate how long the cavity takes to arrive at equilibrium, and at what levels this takes place.

The moisture content of the hygroscopic material at the end of the summer drying period, m'_f, is calculated first. For definiteness, it is assumed that the cavity is enclosed at the beginning of the winter wet-up period and so begins immediately receiving condensation. Referring to Fig. 3, which shows the seasonal cycling of the cavity under steady state conditions, we note first that the summer drying curve is given by equation (20) of the first paper [1] with an initial moisture content given by $m'_f + S$, the peak amplitude of the moisture content during the winter wet-up period; that is

$$m'_f - m_f = (m'_f - m_f + S)\,e^{-t_d/t_2}$$

i.e. $$\Delta m = (\Delta m + S)\delta,$$

where $\delta = e^{-t_d/t_2}$, t_d is the duration of the drying period, t_2 is the cavity drying time constant, see [1], and $\Delta m = m'_f - m_f$ is the excess of moisture concentration of the cavity hygroscopic material in the condensing state over the non-condensing state. t_2 is given by equation (21) in [1] as

$$t_2 = \frac{t_o v}{\mathcal{K}} + t_m.$$

Hence

$$\frac{\Delta m}{S} = \frac{\delta}{1 - \delta},$$

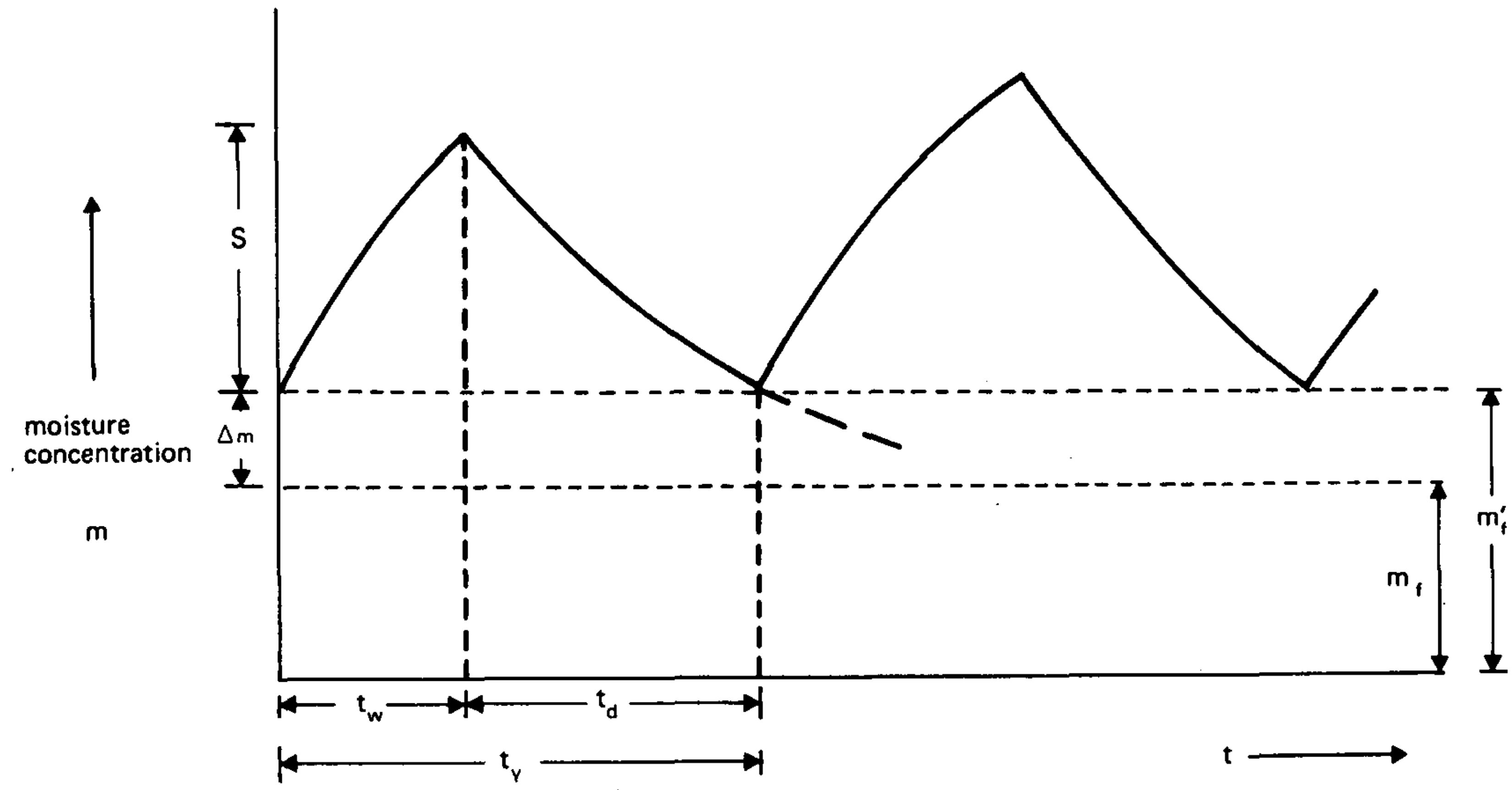

Fig. 3. Condensing cavity in steady state.

so

$$m'_f = m_f + \frac{S\delta}{1-\delta}$$

$$= \frac{\bar{c}}{\mathscr{K}} + \frac{S\delta}{1-\delta} \qquad (6)$$

using formula (18) of the first paper [1].

In turn, this implies that

$$\text{Winter peak moisture content} = m_f + \frac{S}{1-\delta}. \qquad (7)$$

If the drying and wetting curves are assumed roughly linear, the yearly mean moisture content is just the mean of the maximum and minimum moisture contents; i.e.

$$\text{Yearly mean moisture content} = m_f + \frac{S(1+\delta)}{2(1-\delta)}.$$

The term $\delta/(1-\delta)$ is very small for cavities in the hygroscopically controlled drying regime, implying that there is little difference between the summer-end condensing and non-condensing moisture contents in this case. However, if the cavity is in a tighter drying regime, t_2 will be larger implying $\delta/(1-\delta)$ could be of the order of 1. If, for example, the hygroscopic material is wood, the final summer-end moisture content may be boosted a significant 2–3% by weight over the non-condensing case. On the other hand, as previously seen, the amount of winter condensation, S, is smaller for tight cavities. The winter peak, therefore, will not be as high as that for a less tight cavity in a similar environment.

Approach to equilibrium

The non-steady state behaviour of the condensing cavity is now examined, see Figs 1 and 2, and its approach to the steady state examined above is analysed.

If the cavity is enclosed at the beginning of the winter wet-up period with an initial moisture content of M_o, its moisture content at the end of the wet-up period will be $M_o + S$. The cavity will then dry through the summer, the drying being governed by equation (20) of [1], with $M_o + S$ being an initial condition. Using equation (20) of [1], the moisture content of the cavity hygroscopic material at the end of the drying period is given by

$$m = m_f + (M_o + S - m_f)\delta.$$

The process continues year by year, as illustrated in Figs 1 or 2, and it can be shown that at the end of the pth drying period, the moisture content m_p is

$$m_p = m_f + (M_o - m_f)\delta^p + S\sum_{i=1}^{p} \delta^i.$$

Summing the geometric series and using the various definitions, we find

$$\frac{m_p - m'_f}{M_o - m'_f} = \delta^p, \qquad (8)$$

where m'_f is the condensing state steady state value at the end of the drying period, see Fig. 3.

But in the non-condensing state, from equation (20) of [1], at the end of the qth summer the moisture content m_q is

$$\frac{m_q - m_f}{M_o - m_f} = \delta'^q \qquad (9)$$

where $\delta' = e^{-t_y/t_2}$ and t_y is the number of seconds in a year.

Both equations (8) and (9) show the approach of m_p and m_q to their equilibrium values m'_f and m_f respectively, as δ^p and δ'^q tend to zero as p and q tend to infinity. In order for both cases to be equally close to their final state, we require

$$\delta^p = \delta'^q,$$

i.e.

$$\frac{pt_d}{t_2} = \frac{qt_y}{t_2},$$

i.e.

$$p = \frac{t_y q}{t_d}. \qquad (10)$$

Equation (10) states that a cavity in a condensing climate takes longer to approach its equilibrium than the same cavity in a non-condensing climate in inverse proportion to the drying period; the drying period being all year round in a non-condensing climate, and spring, summer and autumn only in a condensing climate. For example, if a cavity is in a climate giving a winter wet-up period of three months, the drying period is then nine months, 3/4 of a year, so the time to equilibrium is 4/3 of the time for the cavity in a non-condensing environment.

The actual time taken to reach equilibrium depends of course on how near to equilibrium is judged to be close enough. This can be defined by requiring the end of drying period moisture level to be within a certain fraction ε of its final value. This requires:

$$\delta^p < \varepsilon,$$

i.e.

$$p > \frac{t_2}{t_d} \ln\left(\frac{1}{\varepsilon}\right). \tag{11}$$

For example, if equilibrium is defined to be reached when the end of drying period moisture concentration is within 10% of its final value, and a cavity is drying under a tighter regime with $t_2 = 1$ year, $t_d = 6$ months $= 0.5$ year, then 4.6 years are required to dry the cavity.

3. EXAMPLES

The following examples, illustrating moisture behaviour, relate to a series of trials on remedial measures being carried out on brick veneer houses in Invercargill, New Zealand [3].

(1) A timber framed house has a pitched roof with leakage rates to and from the exterior atmosphere of 1 air change per hr. Vapour diffusion is negligible in comparison. Relevant parameters are:

Timber volume to cavity volume ratio,
$v = 0.01$.
Area to volume ratio for the pitched roof,
$\frac{A_i}{V_o} = 1\ \mathrm{m}^{-1}$ $(i = 1, 2)$.
Effective R value of the ceiling,
$R_1 = 1.4\ \mathrm{m}^2\ {}^\circ\mathrm{C\ W}^{-1}$.
Effective R value of the roof,
$R_2 = 0.5\ \mathrm{m}^2\ {}^\circ\mathrm{C\ W}^{-1}$.
Mean indoor temperature $\theta_1 = \bar{\theta}_1 = 18°\mathrm{C}$.
Indoors to roof cavity air leakage rate, $F_{10} = 0\ \mathrm{hr}^{-1}$.
Roof cavity to indoors air leakage rate, $F_{01} = 0\ \mathrm{hr}^{-1}$.
Outdoors to roof cavity air leakage rate, $F_{20} = 1\ \mathrm{hr}^{-1}$.
Roof cavity to outdoors air leakage rate, $F_{02} = 1\ \mathrm{hr}^{-1}$.
Dry density of timber (*pinus radiata*) $= 400\ \mathrm{kg\ m}^{-3}$.
Timber time constant, $t_m = 3$ months.
$\mathscr{K} = 10^{-4}$ (see [1]).
$\mu/v = 15$.

The Invercargill climate has the following statistics:
Effective yearly mean indoor temperature = 18°C.
Yearly mean sol-air temperature = 11.9°C.
Daily mean sol-air temperature on coldest day = −3.9°C.
Mean yearly air temperature, $t_2 = 9.3°\mathrm{C}$.
Mean daily temperature on the coldest day = −1.8°C.
Mean yearly vapour concentration, $c_2 = 7.33 \times 10^{-3}\ \mathrm{kg\ m}^{-3}$.

The moisture content of the timber framing is traced as follows.

Yearly mean cavity temperature (formula (1)) = 13.0°C. (Note that air leakage is significant in determining the cavity temperature in this example.)

Cavity saturated vapour concentration at 13.0°C $= 11.4 \times 10^{-3}\ \mathrm{kg\ m}^{-3}$.

Mean cavity temperature on coldest day (formula (1)) = 1.4°C.

Cavity saturated vapour concentration at 1.4°C $= 5.36 \times 10^{-3}\ \mathrm{kg\ m}^{-3}$.

Yearly mean saturated vapour concentration in the cavity,
$c_a = 11.4 \times 10^{-3}\ \mathrm{kg\ m}^{-3}$.

Minimum daily saturated vapour concentration in cavity,
$c_a - c_b = 5.36 \times 10^{-3}\ \mathrm{kg\ m}^{-3}$ (see equation (2)),
$c_b = 6.04 \times 10^{-3}\ \mathrm{kg\ m}^{-3}$,

$$\frac{1}{t_o} = \sum_i \left(\frac{R\bar{T}A_i}{WV_o r_i} + F_{oi}\right) \text{(equation (7), [1])}$$

$\simeq F_{02}$ since air change rate dominates over vapour diffusion

$\therefore\ t_o = 3600$ s,

$$t_2 = \frac{v t_o}{\mathscr{K}} + t_m = 8.24 \times 10^6\ \mathrm{s},$$

(equation (21), Paper 1)

$$\bar{c} = t_o \sum_i \left(\frac{R\bar{T}}{WV_o}\frac{A_i}{r_i} + F_{io}\right) c_i$$

(equation (13), Paper 1)

$$\simeq t_o F_{20} c_2$$

$$= c_2 = 7.33 \times 10^{-3}\ \mathrm{kg\ m}^{-3}.$$

Take the winter mean vapour concentration as equal to the final long-term equilibrium vapour concentration,

i.e. $\bar{c}_w = c_f$.

But formula (19), Paper 1 [1] shows that $c_f = \bar{c}$ and since $\bar{c}$ has been calculated as $7.33 \times 10^{-3}\ \mathrm{kg\ m}^{-3}$, then $\bar{c}_w = 7.33 \times 10^{-3}\ \mathrm{kg\ m}^{-3}$.

Therefore the mean long term cavity conditions are:
Cavity temperature = 13.0°C.
Cavity vapour concentration $= 7.33 \times 10^{-3}\ \mathrm{kg\ m}^{-3}$.
Corresponding cavity relative humidity = 65%.
Corresponding timber equilibrium moisture content [5] = 12.2% by weight = $48.8\ \mathrm{kg\ m}^{-3}$.

Winter accumulated timber moisture concentration (equation (5))

$$S = 45.1\ \mathrm{kg\ m}^{-3},$$

i.e. winter accumulated timber moisture content = 11.3% by weight.

Duration of the winter wet-up period (equation (3)),

$$t_w = 0.265 \text{ year}.$$

Duration of the summer drying period = 0.735 year.

$$\delta = e^{-t_d/t_2} = 0.060.$$

Timber moisture content at the end of the summer drying period (equation (6))

$$m'_f = 51.6\ \mathrm{kg\ m}^{-3} \text{ or } 12.9\% \text{ by weight}.$$

Timber moisture content at the end of the winter wet-up period $= m'_f + S = 51.6 + 45.1 = 96.7\ \mathrm{kg\ m}^{-3}$ or 24.2% by weight.

Yearly mean moisture content = 18.6% by weight.

(2) A common problem with brick veneer houses in New Zealand is that the wall cavity sometimes provides an uninterrupted flow path of relatively warm moist air from the subfloor space into the roof space. This example illustrates that this moist air seriously increases condensation problems, especially if the subfloor space is ponded with surface water.

It is assumed that air at 11°C and 90% relative humidity flows from the subfloor space into the roof cavity at the rate of 0.3 air changes per hour. The roof cavity air still leaves at 1 air change per hr, implying that the external air mixes into the roof cavity at a rate of 0.7 air changes per hr.

Parameters that have changed from example (1) are:

$$\left.\begin{array}{l} F_{20} = 0.7\ \text{hr}^{-1} \\ F_{30} = 0.3\ \text{hr}^{-1} \\ F_{03} = 0\ \text{hr}^{-1} \end{array}\right\} \text{ where region 3 is the subfloor space.}$$

Calculating as before, we find:

Yearly mean cavity temperature = 13.0°C.

Cavity saturated vapour concentration at 13.0°C $= 11.4 \times 10^{-3}$ kg m^{-3}.

Mean cavity temperature on the coldest day = 1.9°C.

Cavity saturated vapour concentration at 1.9°C = 5.55 $\times 10^{-3}$ kg m^{-3}.

$$c_a = 11.4 \times 10^{-3}\ \text{kg m}^{-3}.$$

$$c_b = 5.85 \times 10^{-3}\ \text{kg m}^{-3}.$$

$$\frac{1}{t_o} \simeq F_{02}, \quad \text{as before.}$$

$$\therefore t_o = 3600\ \text{s}.$$

$$t_m = 7.88 \times 10^6\ \text{s}, \quad \text{as before.}$$

$$t_2 = \frac{vt_o}{\mathscr{K}} + t_m = 8.24 \times 10^6\ \text{s}, \quad \text{as before.}$$

$$\bar{c} \simeq t_o(F_{20}c_2 + F_{30}c_3) = 7.85 \times 10^{-3}\ \text{kg m}^{-3}.$$

Take $\bar{c}_w = c_f = \bar{c} = 7.85 \times 10^{-3}$ kg m^{-3}.

Therefore the mean long term cavity conditions are:

Cavity temperature = 13.0°C.

Cavity vapour concentration $= 7.85 \times 10^{-3}$ kg m^{-3}.

Corresponding cavity relative humidity = 70%.

Corresponding timber emc = 13.4%.

Winter wet-up vapour absorbed = 58.1 kg m^{-3},

i.e. 14.5% moisture content:

$$t_w = 0.292\ \text{year},$$

$$\delta = 0.067.$$

Hence moisture content at the end of drying period = 14.4%.

Moisture content at the end of winter wet-up period = 28.9%.

Mean moisture content = 21.7%.

(3) One apparently effective remedial measure to overcome the problem of excessive moisture (at present under trial by BRANZ) is to place polythene sheet on the ground in the subfloor space. In this case, the vapour concentration of the air from the subfloor cavity will be approximately equal to the vapour concentration in the external air; i.e. $c_3 = 7.33 \times 10^{-3}$ kg m^{-3}. All other parameters remain the same as in example (2). The effectiveness of this remedial measure is demonstrated in the following calculations of the timber moisture content.

Yearly mean cavity temperature = 13.0°C.

Mean cavity temperature on coldest day = 1.9°C.

$$c_a = 11.4 \times 10^{-3}\ \text{kg m}^{-3}.$$

$$c_b = 5.85 \times 10^{-3}\ \text{kg m}^{-3}.$$

$t_o = 3600$ s, $t_m = 7.88 \times 10^6$ s, $t_2 = 8.24 \times 10^6$ s, as before.

$$\bar{c} = t_o(F_{20}c_2 + F_{30}c_3) = 7.33 \times 10^{-3}\ \text{kg m}^{-3}.$$

Take $\bar{c}_w = c_f = \bar{c} = 7.33 \times 10^{-3}$ kg m^{-2}.

Therefore the mean long term cavity conditions are:

Cavity temperature = 13.0°C.

Cavity vapour concentration $= 7.33 \times 10^{-3}$ kg m^{-3}.

Corresponding cavity relative humidity = 65%.

Corresponding timber emc = 12.2%.

Winter wet-up vapour absorbed = 39.34 kg m^{-3},

i.e., 9.8% moisture content:

$$t_w = 0.255\ \text{year},$$

$$\delta = 0.058.$$

Hence moisture content at the end of the drying period = 12.8%.

Moisture content at the end of the winter wet-up period 22.6%.

Mean moisture content = 17.7%.

The results of the above calculations concur well with the results observed under the BRANZ remedial measure programme [3], and provide encouraging preliminary verification of the validity of the theoretical approach undertaken in this paper.

4. RECOMMENDATIONS

In addition to the four recommendations made to the designer in the first paper, another can now be added, viz:

(5) If a tighter drying regime is chosen, viz the intermediate or construction controlled regimes, designers should use the formula for t_2, equation (21) in [1] plus equation (3) and (11) in this paper to calculate the length of the winter wet-up period and consequent modification of the time to equilibrium. In unfavourable conditions this can be many years. Under no circumstances should very wet timber be enclosed into a cavity that will dry under these tighter regimes.

5. CONCLUSIONS

This paper has outlined an approach to link the moisture behaviour of a cavity in a condensing environment to that in a non-condensing environment, through the use of simple climate statistics, and in particular has shown the connection between their steady state moisture contents, and their times to equilibrium.

Many of the results derived in these two papers may appear obvious in retrospect. That this is so is a tribute to the depth of experience and soundness of engineering judgement accumulated over the years of dealing with moisture problems. Nevertheless, an approach such as undertaken in these two papers provides a sound physical basis for explaining field conditions, allowing predictions of the consequences of remedial measures, and providing the basis for an accurate design tool.

The quantitative aspects of this work are not yet backed by a detailed enough explanation and quantification of some physical processes, particularly in explaining what happens to the condensation, to lay any claim to it being a final design tool. Consequently, only preliminary validation against experimental and field data has so far been attempted. However, the same remarks can be said for earlier design methods, such as the dew point profile technique and the Keiper method, both of which are based on a less complete physical description than outlined in this paper. A research project [3] is in hand to provide some of these details. Indeed, this work arose out of preliminary studies to background the performance of a numerical model the author is developing to explain and predict experimental and field data being collected on the moisture performance of rooves.

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