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Modelling of some dwelling internal microclimates

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Abstract

The microclimates of dust mite microhabitats in the base of carpets and in bedding are modelled analytically. The non-dimensional parameters characterising these systems are identified and analytical solutions are presented using these non-dimensionless groups. These results are compared to experimental results derived using a recently developed very small relative humidity sensor. With this sensor it has been possible to distinguish between temperatures and humidities in microenvironments separated only by millimetres, e.g. microclimates in the base and the top of a carpet, or microclimates above and below a sheet in a bed. Despite linearity assumptions and other simplifications, good agreement is found between experimental and theoretical results.

Nomenclature

$\mathbb{B}$, $\mathbb{C}$, $\mathbb{D}$, $\mathbb{H}$, $\mathbb{K}$, $\mathbb{L}$ various dimensionless constants [see eqns (52)–(57)]
D coefficient of moisture diffusion under vapour pressure driving forces (s)
d scale distance (m)
H surface heat transfer coefficient (W m^{-2} °C^{-1})
h surface mass transfer coefficient under vapour pressure driving forces (s m^{-1})
j unit imaginary number
k thermal conductivity (W m^{-1} °C^{-1})
l element dimension (m)
m moisture content (kg m^{-3})
$\mathbb{P}$ dimensionless vapour pressure [eqn (12)]
p vapour pressure (Pa)
$\bar{p}$ mean value of vapour pressure (Pa)
$\mathbb{T}$ dimensionless temperature [eqn (4)]
T temperature (°C)
$\bar{T}$ mean value of temperature (°C)
t time (s)
t_p time constant for moisture transfer [eqn (12)] (s)
t_T time constant for heat transfer [eqn (4)] (s)
X dimensionless distance [eqn (4)] (s)

Greek symbols
α diffusivity (W m^2 J^{-1})
β diffusivity for moisture diffusion (Pa m^3 s kg^{-1})
$\Delta\mathbb{T}$ dimensionless quantity [see eqn (50)]
Δp, ΔT, $\Delta\Phi$ amplitude of the daily cyclic component of the quantity
Φ relative humidity
ρ density (kg m^{-3})
τ_p dimensionless time in units of t_p [eqn (12)]
τ_T dimensionless time in units of t_T [eqn (4)]
ω angular frequency corresponding to a period of one day (7.272×10^{-5} radians s^{-1})

Subscripts
$c\Phi$ associated with moisture transfer in the carpet
$f\Phi$ associated with moisture transfer in the floor

1. Introduction

1.1. Purpose of work

It is the purpose of this work to explain recently acquired results [20] of temperature, relative humidity and vapour pressure collected in the microhabitats of biocontaminants in dwellings. By analytical modelling we seek to capture the essential features of the, often complex, changes in microclimates over 24-h periods. Appropriate dimensionless groups are sought which characterise the systems under investigation and which will remain relevant when more accurate numerical modelling is undertaken.

1.2. Biocontaminant control and microclimates

Amongst the contaminants affecting the indoor environment, two biocontaminants are of particular

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interest, viz dust mites and moulds. These biocontaminants are widespread and pose particular problems.

Dust mite faeces contain the leading allergen, Der p 1, for the development and provocation of asthma [1–5] and as such, many groups around the world have studied ways of reducing dust mite populations [6–13]. Dust mites absorb moisture hygroscopically from the air and so cannot survive if humidities become too low [14–16], below the level known as the critical equilibrium humidity (CEH). Consequently, one method of dust mite control is to lower room humidities, aiming to make conditions too dry in the dust mites' microhabitat [11, 12, 17].

Moulds affect material durability, present a health hazard to those allergic to their spores or metabolic by-products [18], and, being unsightly, reduce property values. As with dust mites, moulds of concern in buildings also require higher humidities to be viable [19].

These biocontaminants inhabit distinctive microenvironments of millimetre scale. Moulds grow on surfaces that attract condensation or whose local relative humidity is likely to be at least 80%, such as thermal bridges, behind furniture placed against exterior walls, window frames, etc. Dust mites are found in bedding, the base of carpets and in furniture.

In all of these microenvironments the microclimate is often quite different from room ambient conditions. Failure to appreciate the difference between room conditions and microenvironmental conditions is one factor leading to the mixed success of attempts to control dust mite populations by controlling room conditions [20].

Room conditions are but one factor influencing microenvironmental climates. Very important also are: the hygrothermal parameters of the surrounding materials, such as their heat and moisture diffusivities and absorption properties, be they building envelope elements like walls and floors, or bedding, furniture, fabrics, etc.; the presence of local heat and moisture sources, such as a person in bed or using furniture; and, particularly for building envelope elements, external psychrometric conditions.

As a consequence, attempts to influence biocontaminant viability by changing room psychrometric conditions through heating, ventilating, dehumidifying, etc., may or may not result in a change in microclimate that effectively controls biocontaminants.

2. Work to date

The author has developed a relative humidity sensor that allows discrimination of differences in humidities in locations separated by millimetres [21]. This has allowed humidities in the base of carpets to be measured and discriminated from values measured on the top of the carpet, or has allowed humidity sensors to be placed through layers of bedding under a bed occupant, and discriminate between humidities immediately beneath the occupant from humidities just millimetres away under sheets, above and below electric blankets, on mattresses, etc. [20]. Figures 1 and 2 illustrate the nature of these results.

Figure 1 shows relative humidities, temperatures and vapour pressures at various levels through bedding over 24 h. Figure 2 shows relative humidity, temperature and vapour pressure changes in a room and at the base of the carpet over 48 h.

Results for humidities under a bed occupant, Fig. 1, are complex and, at first sight, perhaps counter-intuitive. Most significantly, when the bed is occupied, relative humidities fall under the bottom sheet just a millimetre or so from the occupant. The same is true under the electric blanket (never switched on) on top of the mattress and under the bottom sheet. It is not until deeper in the mattress that relative humidities are observed to rise. When the bed is vacated, conditions return to room conditions with a time constant of several hours.

Results for the carpet, Fig. 2, are more straightforward. Here it is observed that base-of-carpet humidities have a higher mean than room humidities, have a smaller daily amplitude, and are phase-shifted compared to the room humidity.

These results are explained here by solving the differential equations of heat and mass transfer describing these two scenarios. These solutions are then compared to experimental data. The models are given in dimensionless form so that the essential parametric groups describing and predicting the microclimates are uncovered.

3. Bedding case

3.1. Assumptions and simplifications

In this case the transient temperature, vapour pressure, and humidity response below a bed occupant are modelled for the time period immediately after the bed is occupied.

The bedding under a bed occupant is modelled as one material of finite depth, i.e. no distinction is made between the mattress and the sheets, electric blankets or any other layers on top of the mattress and immediately below the occupant, see Fig. 3. Boundary conditions at the top of the bedding at time zero are stepped up to an assumed fixed value determined by human physiology. Boundary conditions at the bottom of the mattress are assumed to be constant throughout.

Inspection of the experimental data collected [20] show that these are reasonable assumptions.

In this work it is assumed that vapour pressure is the dominant moisture-driving potential, so that the differential equation governing moisture transfer is

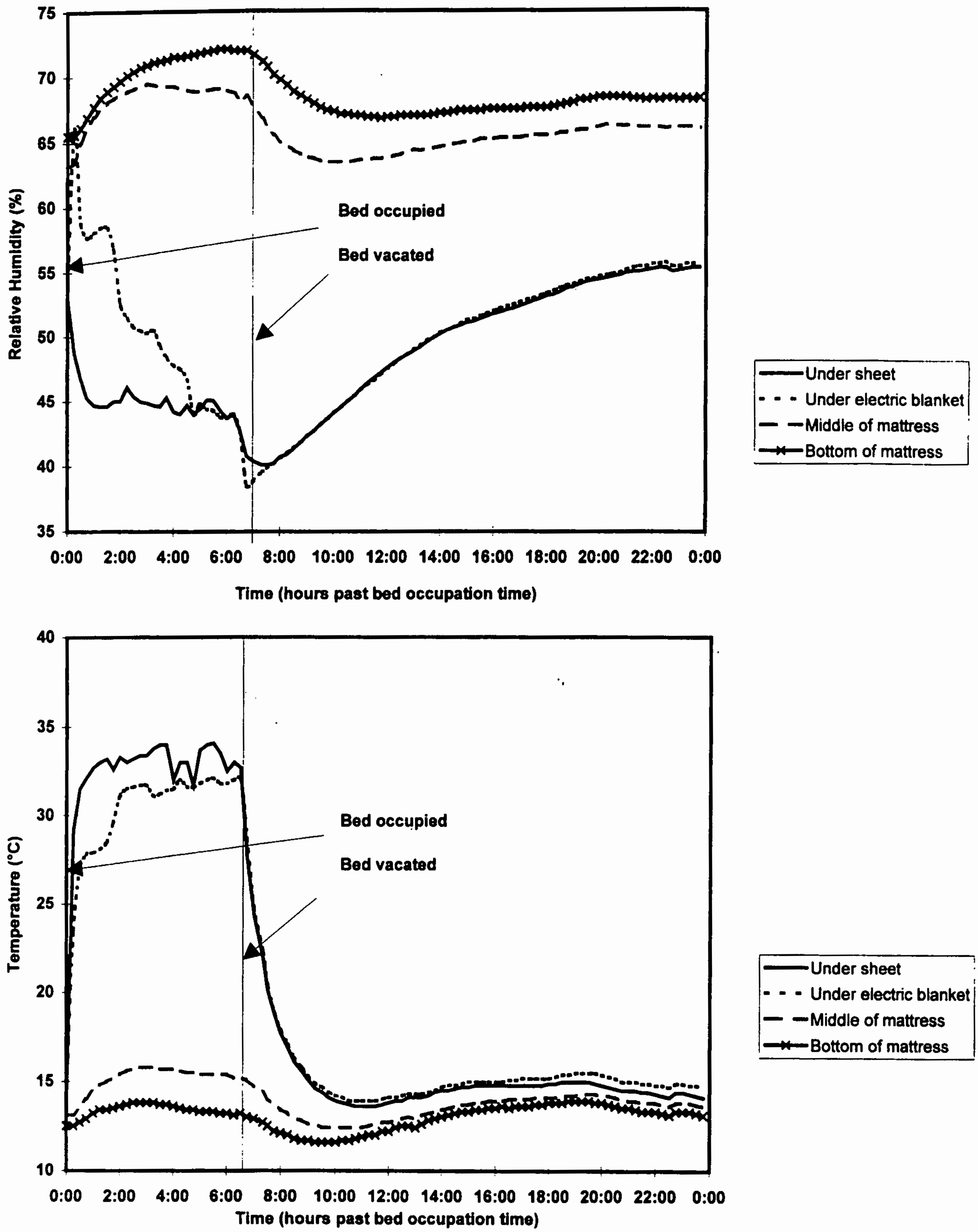


Fig. 1. Psychrometric conditions in bedding below the occupant. At levels close to the occupant, relative humidities rise when the bed is occupied; at deeper levels relative humidities fall. Temperatures immediately below the occupant rise close to blood temperature of 37°C, while vapour pressures become very high.

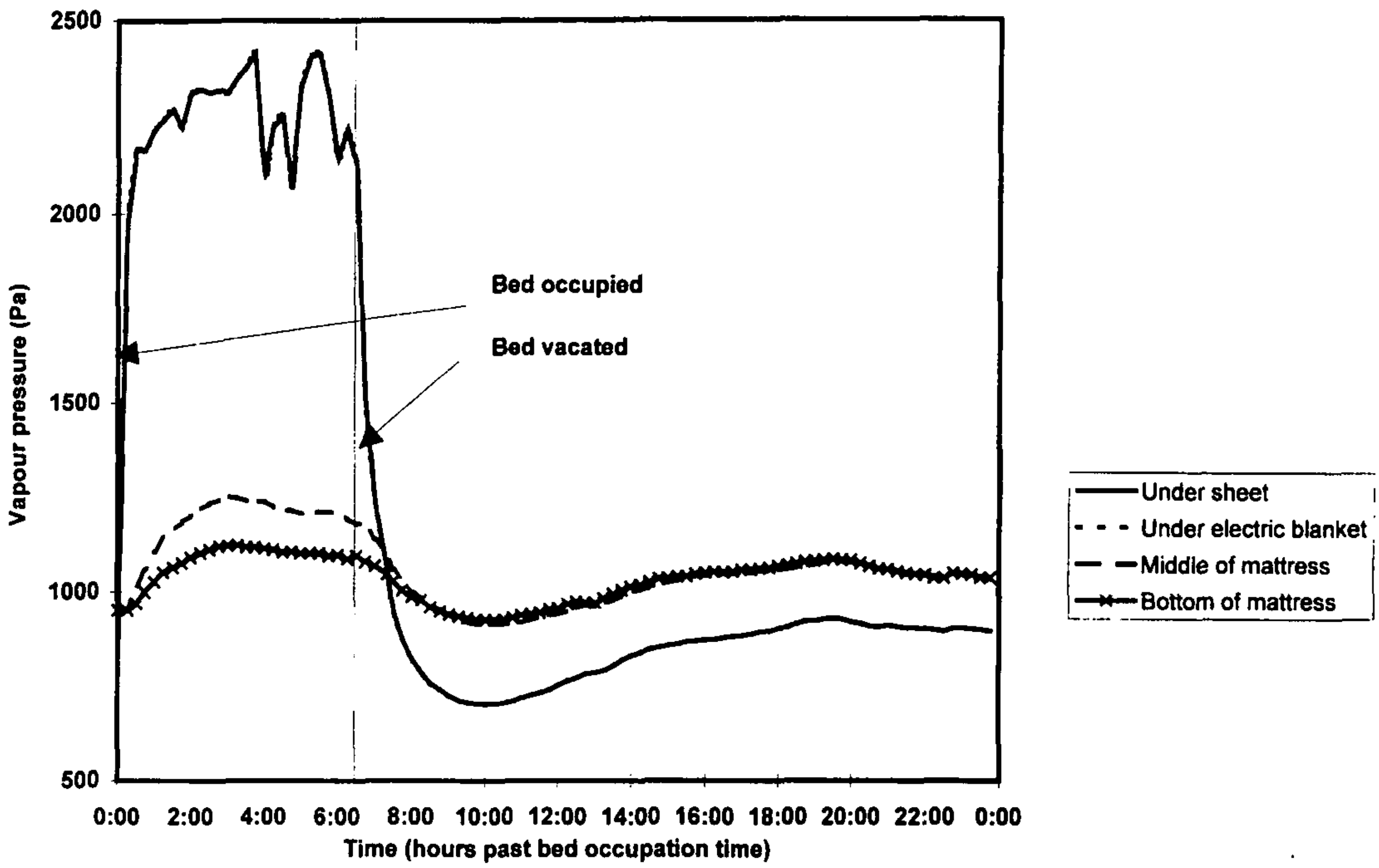


Fig. 1—*continued.*

$$\frac{\partial m}{\partial t} = D\frac{\partial^2 p}{\partial x^2} \qquad (1)$$

where D is the diffusion coefficient for vapour pressure-driven moisture transfer, while for heat transfer we have

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c}\frac{\partial^2 T}{\partial x^2} = \alpha\frac{\partial^2 T}{\partial x^2} \qquad (2)$$

using standard notation.

Dimensionlessly, eqn (2) becomes

$$\frac{\partial \mathbb{T}}{\partial \tau} = \frac{1}{\pi^2}\frac{\partial^2 \mathbb{T}}{\partial X^2} \qquad (3)$$

where

$$\mathbb{T} = \frac{T - T_{\text{bottom}}}{T_{\text{end}} - T_{\text{bottom}}}, \quad X = x/l, \quad t_T = \frac{1}{\pi^2}\frac{l^2}{\alpha}, \quad \tau_T = t/t_T \qquad (4)$$

T_{start} (see below) is the temperature on the top surface of the mattress before it is occupied, and T_{end} is the temperature established there, taken as constant, the moment after the bed is occupied. T_{bottom} is the temperature on the bottom surface of the mattress taken as constant throughout the process. l is the thickness of the mattress and x is the distance into the mattress from the top.

It has been found that when the occupant leaves the bed the mattress cools, but provided that the covers are not pulled back, the cooling process has not finished when the bed is re-occupied the next evening. Hence roughly linear temperature and vapour pressure gradients exist within the mattress and sheets when the bed is re-occupied. This situation will be mirrored in the model by assuming linear temperature and vapour pressure gradients as the initial conditions, i.e.

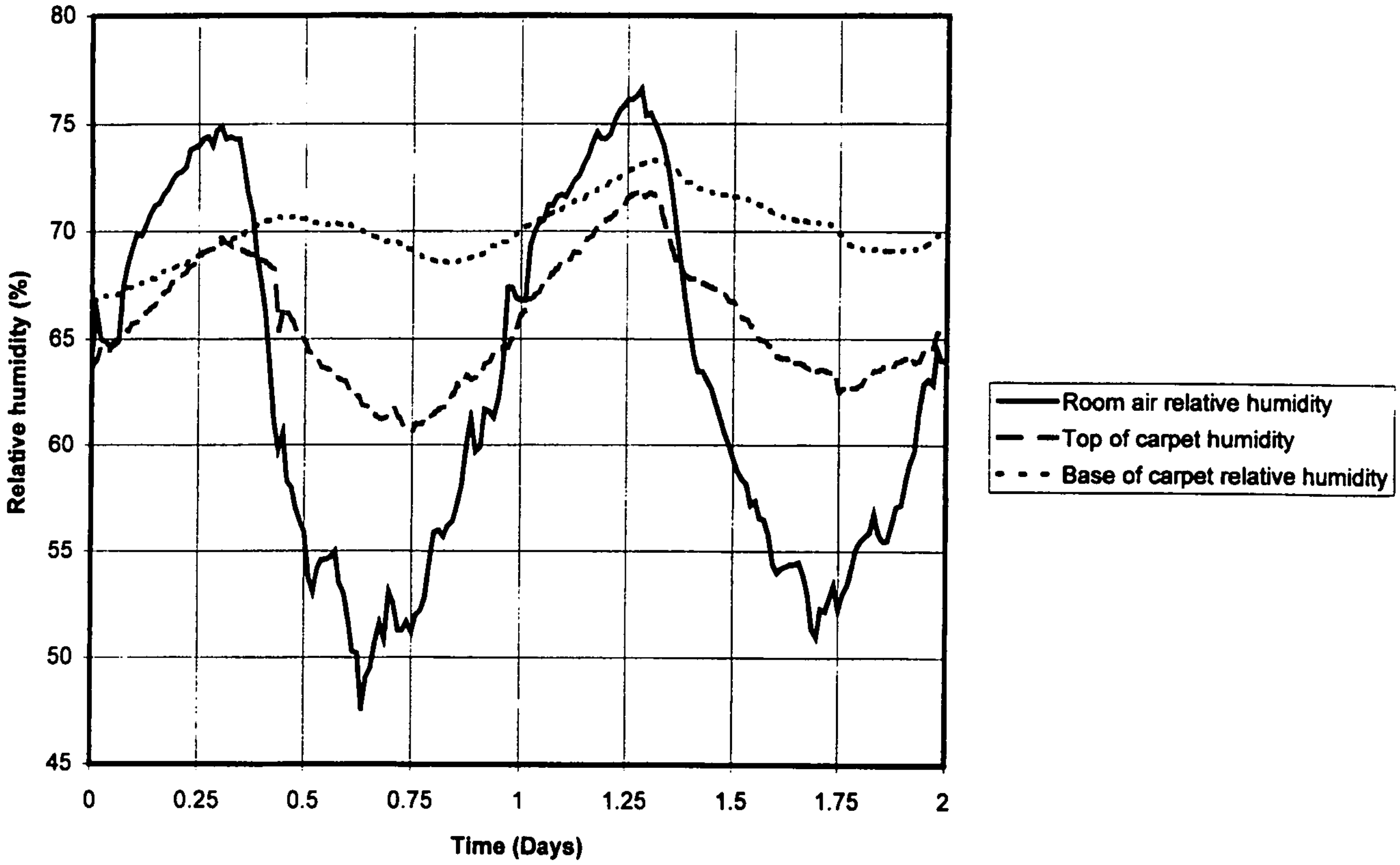
$$T(t=0) = T_{\text{start}} - (T_{\text{start}} - T_{\text{bottom}})\frac{x}{l}$$

and

$$p(t=0) = p_{\text{start}} - (p_{\text{start}} - p_{\text{bottom}})\frac{x}{l} \qquad (5)$$

The boundary conditions used then are

$$T(x=0) = T_{\text{start}}, \quad T(x=l) = T_{\text{bottom}} \quad t < 0$$
$$p(x=0) = p_{\text{start}}, \quad p(x=l) = p_{\text{bottom}} \quad t < 0$$
$$T(x=0) = T_{\text{end}}, \quad T(x=l) = T_{\text{bottom}} \quad t > 0$$
$$p(x=0) = p_{\text{end}}, \quad p(x=l) = p_{\text{bottom}} \quad t > 0 \qquad (6)$$



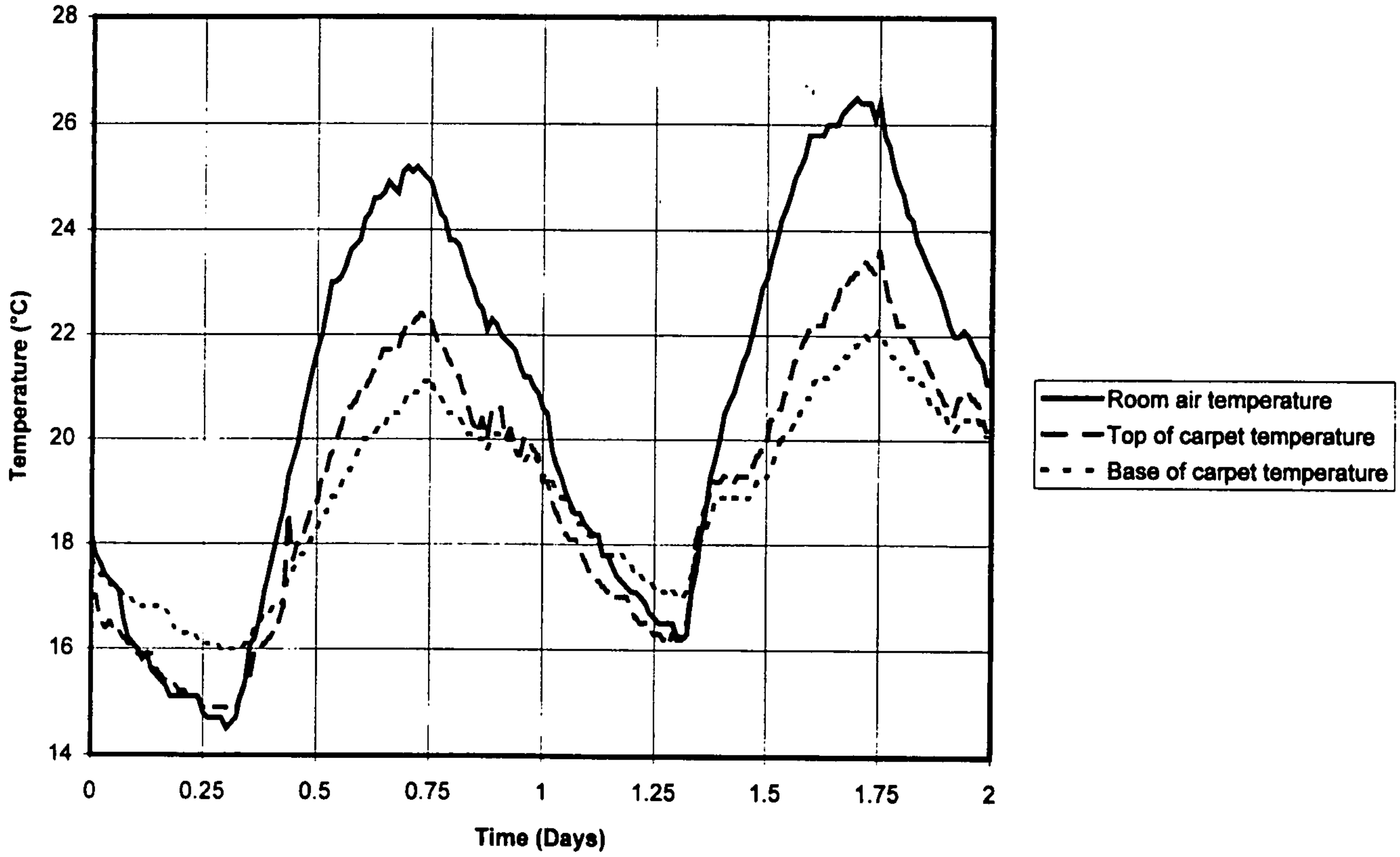


Fig. 2. Measured room and carpet psychrometric conditions. Humidities in the base of the carpet are higher than room levels, and more damped.

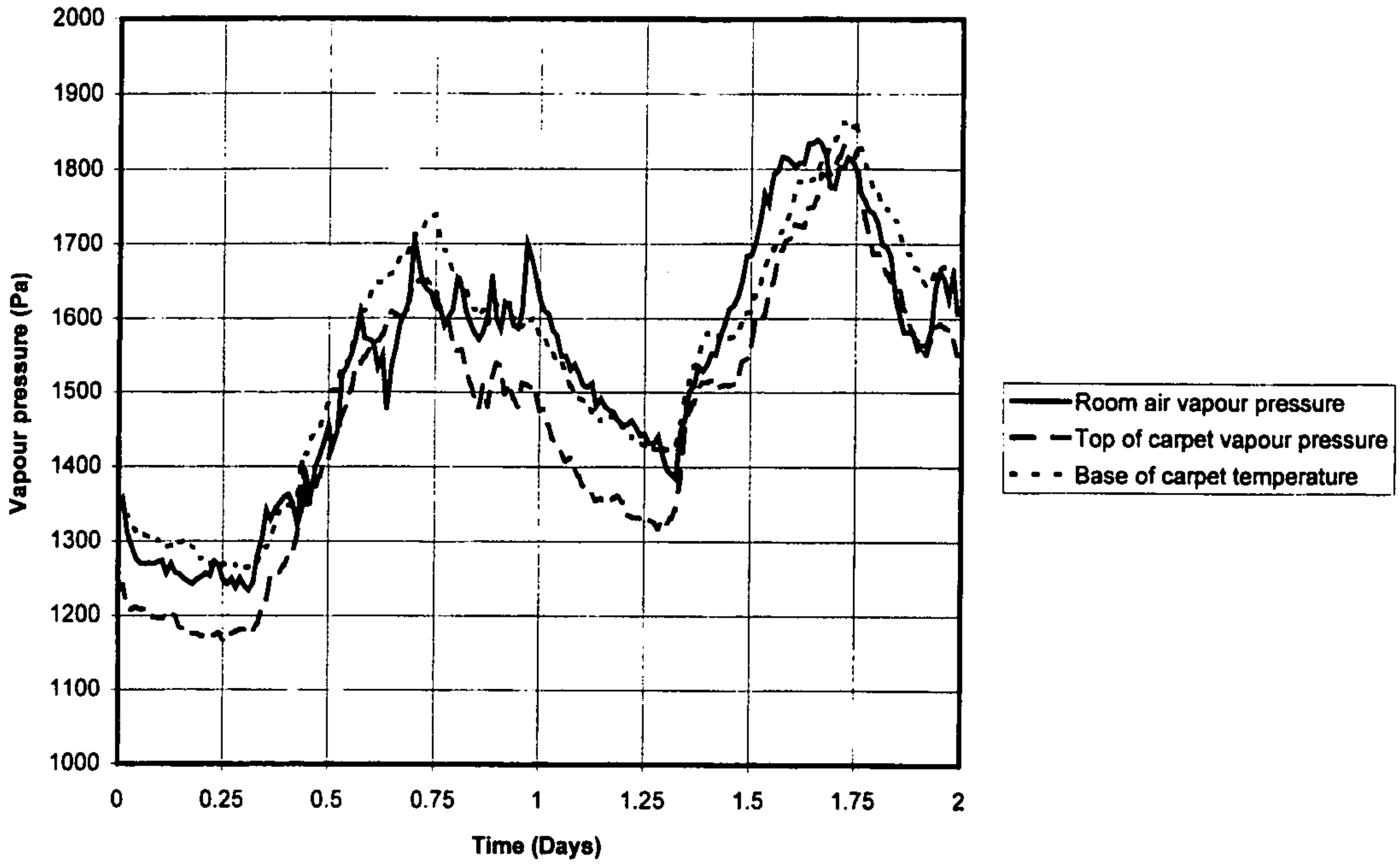


Fig. 2—*continued.*

3.2. Solutions for the bedding case

Standard methods, see for example Carslaw and Jaeger [21], give the temperature solution to eqn (2) subject to the initial and boundary conditions of eqns (5) and (6) as

$$T = T_{\text{end}} - \frac{(T_{\text{end}} - T_{\text{bottom}})x}{l} + (T_{\text{end}} - T_{\text{start}})\frac{2}{\pi}\sum_{n=1}^{\infty}(-1)^n \times \exp\left(-\frac{\alpha n^2\pi^2 t}{l^2}\right)\sin\left(\frac{n\pi(1-x)}{l}\right) \quad (7)$$

or dimensionlessly

$$\mathbb{T} = 1 - X + \frac{2}{\pi}(1 - \mathbb{T}_{\text{start}})\sum_{n=1}^{\infty}(-1)^n \exp(-n^2\tau_T) \times \sin(n\pi(1-X)) \quad (8)$$

where

$$\mathbb{T}_{\text{start}} = \frac{T_{\text{start}} - T_{\text{bottom}}}{T_{\text{end}} - T_{\text{bottom}}} \quad (9)$$

Equation (1) can be written as

$$\frac{\partial m}{\partial p}\frac{\partial p}{\partial t} + \frac{\partial m}{\partial T}\frac{\partial T}{\partial t} = D\frac{\partial^2 p}{\partial x^2} \quad (10)$$

Dimensionlessly, this becomes

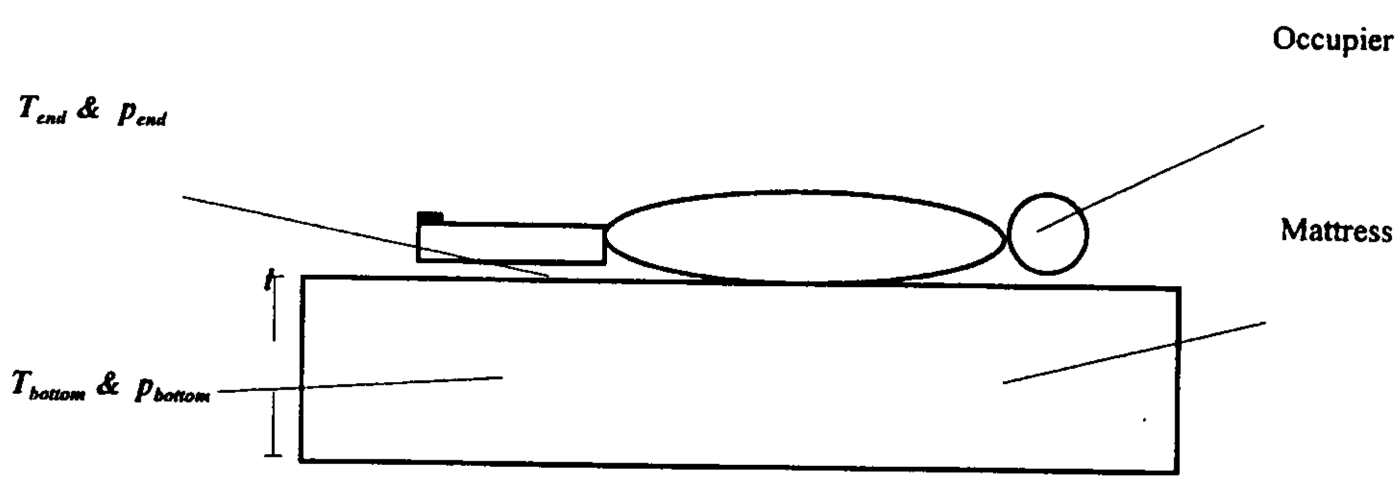


Fig. 3. Geometry and boundary conditions for a mattress.

$$\frac{\partial \mathbb{D}}{\partial \tau_p}+\left(\frac{T_{\text{end}}-T_{\text{bottom}}}{P_{\text{end}}-P_{\text{bottom}}}\right)\left[\frac{\frac{\partial m}{\partial T}}{\frac{\partial m}{\partial p}}\right]\frac{\partial \mathbb{T}}{\partial \tau_p}=\frac{1}{\pi^2}\frac{\partial^2 \mathbb{P}}{\partial X^2} \tag{11}$$

where

$$\mathbb{P}=\frac{p-p_{\text{bottom}}}{p_{\text{end}}-p_{\text{bottom}}},\quad t_p=\frac{1}{\pi^2}\frac{\partial m}{\partial p}\frac{l^2}{D},\quad \tau_p=t/t_p \tag{12}$$

Equation (11) is linearised by taking $\partial m/\partial T$ and $\partial m/\partial p$ as constant, using their average value over the range of temperatures and vapour pressures of interest.

It is equally valid to transform eqn (1) to

$$\frac{\partial m}{\partial \Phi}\frac{\partial \Phi}{\partial t}+\frac{\partial m}{\partial T}\frac{\partial T}{\partial t}=D\left(\frac{\partial p}{\partial \Phi}\frac{\partial^2 \Phi}{\partial x^2}+\frac{\partial p}{\partial T}\frac{\partial^2 T}{\partial x^2}\right) \tag{13}$$

as is done in the carpet case, eqn (35) below (where also $\partial m/\partial T$ is taken equal to zero)—however, the complex behaviour of humidity Φ which we wish to model is due in part to the non-linear behaviour of the saturated vapour pressure of water. Equation (13) evaluates humidity directly, and in light of the linearity simplifications that are being made, the complex behaviour of the humidity will not show itself if eqn (13) forms the basis for solution. On the other hand, solving for temperature and vapour pressure and then calculating relative humidity from these variables preserves the non-linear complex behaviour of humidity which we wish to explore.

The solution to this eqn (11) with the initial conditions of eqn (5) and boundary conditions of eqn (6) takes the form

$$\mathbb{P}=M+\sum_1^{\infty}(A_n\exp(-n^2\tau_p)+B_n\exp(-n^2\tau_T)) \tag{14}$$

where M, A_n and B_n are found by substituting eqn (14) into eqn (11) and using the initial conditions of eqn (5) and boundary conditions of eqn (6).

This yields

$$\mathbb{P}=1-X+(1-\mathbb{P}_{\text{start}}+\mathbb{C}(1-\mathbb{T}_{\text{start}}))\times\frac{2}{\pi}\sum_1^{\infty}\frac{(-1)^n}{n}\exp(-n^2\tau_p)\sin(n\pi(1-X))$$
$$-\mathbb{C}(1-\mathbb{T}_{\text{start}})\frac{2}{\pi}\sum_1^{\infty}\frac{(-1)^n}{n}\exp(-n^2\tau_T)\sin(n\pi(1-X)) \tag{15}$$

where

$$\mathbb{P}_{\text{start}}=\frac{p_{\text{start}}-p_{\text{bottom}}}{p_{\text{end}}-p_{\text{bottom}}} \tag{16}$$

and

$$\mathbb{C}=\left(\frac{T_{\text{end}}-T_{\text{bottom}}}{p_{\text{end}}-p_{\text{bottom}}}\right)\left[\frac{\frac{\partial m}{\partial T}}{\frac{\partial m}{\partial p}}\right]\left(\frac{t_p}{t_p-t_T}\right) \tag{17}$$

Given the temperature, eqn (8), and the vapour pressure, eqn (15), relative humidities can be calculated using standard psychrometric formulae, see for instance the CIBSE handbook [23].

4. Comparison with experimental results

Figure 5 shows the comparison between experimental and calculated relative humidities in the bedding, using eqns (8) and (15). The comparison is illustrative only as actual physical values were not used; rather a fit was made using values in the range to be expected. Values used are listed in Table 1.

It can be seen that the modelled humidities reproduce the measured humidities comparatively well for locations close to the occupant, but not so well deeper in the mattress. The latter difference is almost certainly because the mattress is inner sprung so does not represent well the model assumption of uniform material properties throughout the mattress.

Table 1
Values of quantities used to fit calculated mattress humidities to experimental data

Quantity	Type	Value	Cross reference (equation)
t_p	Fitted	191 min	(12)
t_T	Fitted	95.8 min	(4)
$\mathbb{T}_{\text{start}}$	From experimental data	0.0866	(9)
$\mathbb{P}_{\text{start}}$	From experimental data	−0.0897	(16)
$\frac{T_{\text{end}}-T_{\text{bottom}}}{P_{\text{end}}-P_{\text{bottom}}}$	From experimental data	0.0148°C/Pa	(11)
$\mathbb{C}$	Calculated	−2.09	(17)

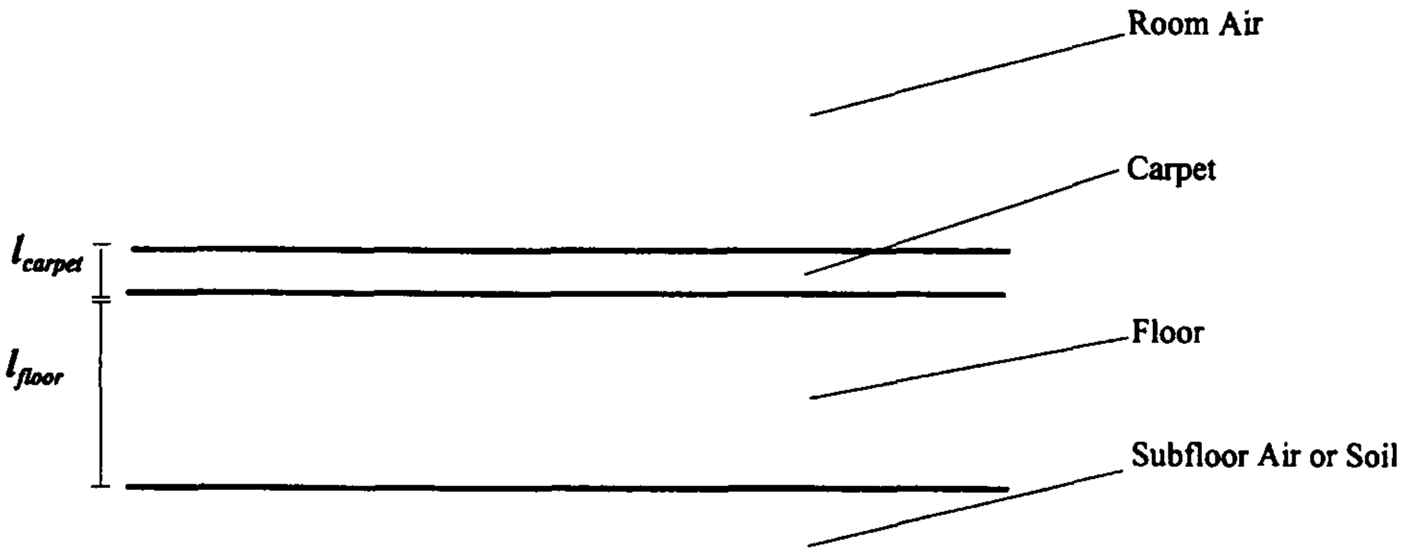


Fig. 4. Geometry of the carpet–flooring system.

5. Carpet case

5.1. Assumptions and simplifications

In this case the daily fluctuations of the microclimate in the base of a carpet are modelled as a function of the daily cycles of temperature and humidity experienced in an uncontrolled room. It is at the base of the carpet where conditions will be most different from those in the room and most likely to be conducive to mite viability, so the derivation below is directed towards finding temperatures and humidities at this location.

The system is modelled as two layers, the carpet and the floor, see Fig. 4. It is assumed that the carpet pile fibres and the air between the pile fibres can be modelled as one homogeneous material. Equivalent heat and moisture diffusivities are chosen to match the performance of the homogeneous modelled carpet to the actual heterogeneous carpet.

Beneath the carpet pile a second layer is specified. Physically this will correspond to the carpet backing, plus any carpet underlay, plus the floor, plus any subfloor insulation. It will be seen that lumping these elements into one is not an unreasonable simplification.

For simplicity this multiple carpet base-floor layer will be called the floor layer in the rest of this work. Physically,

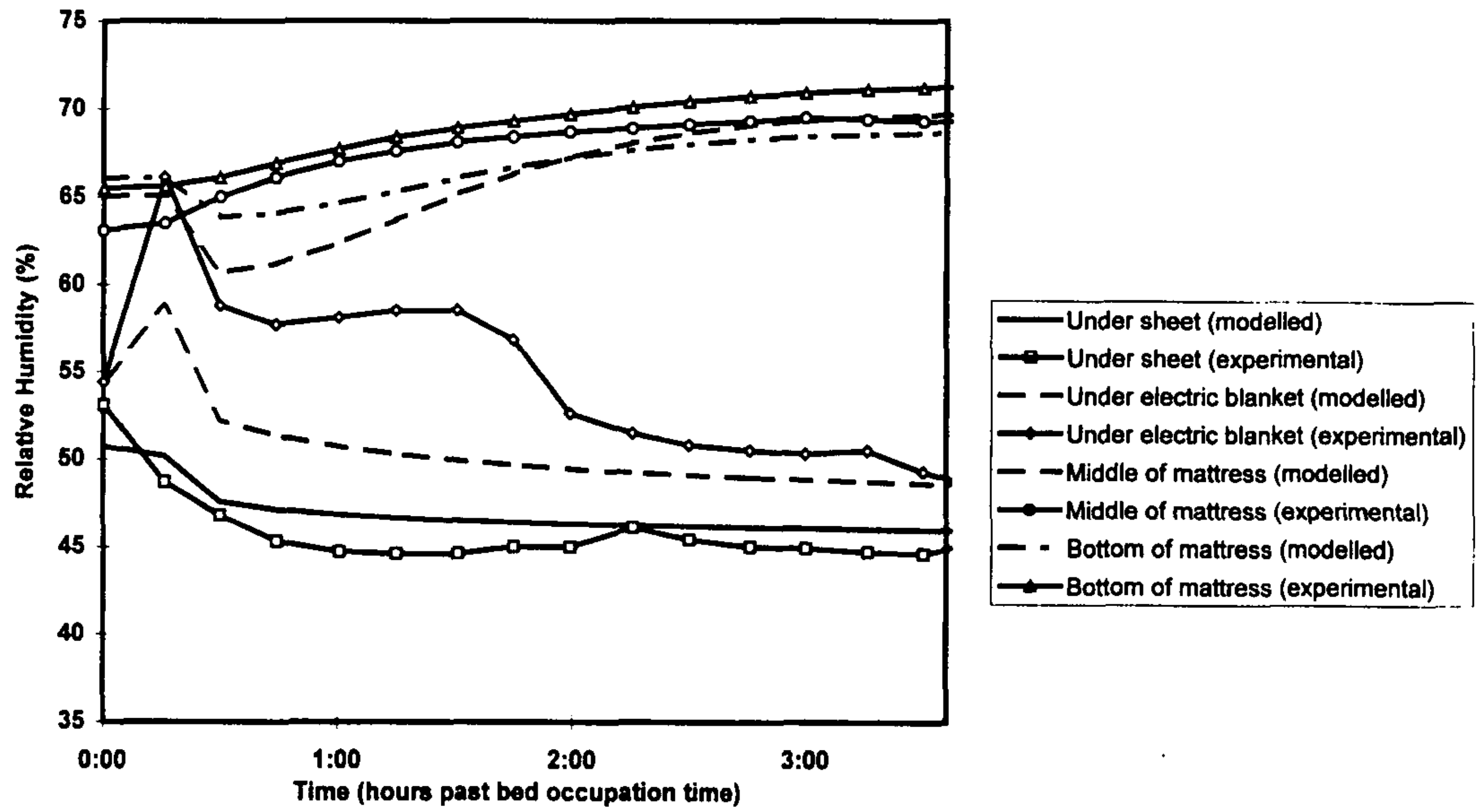
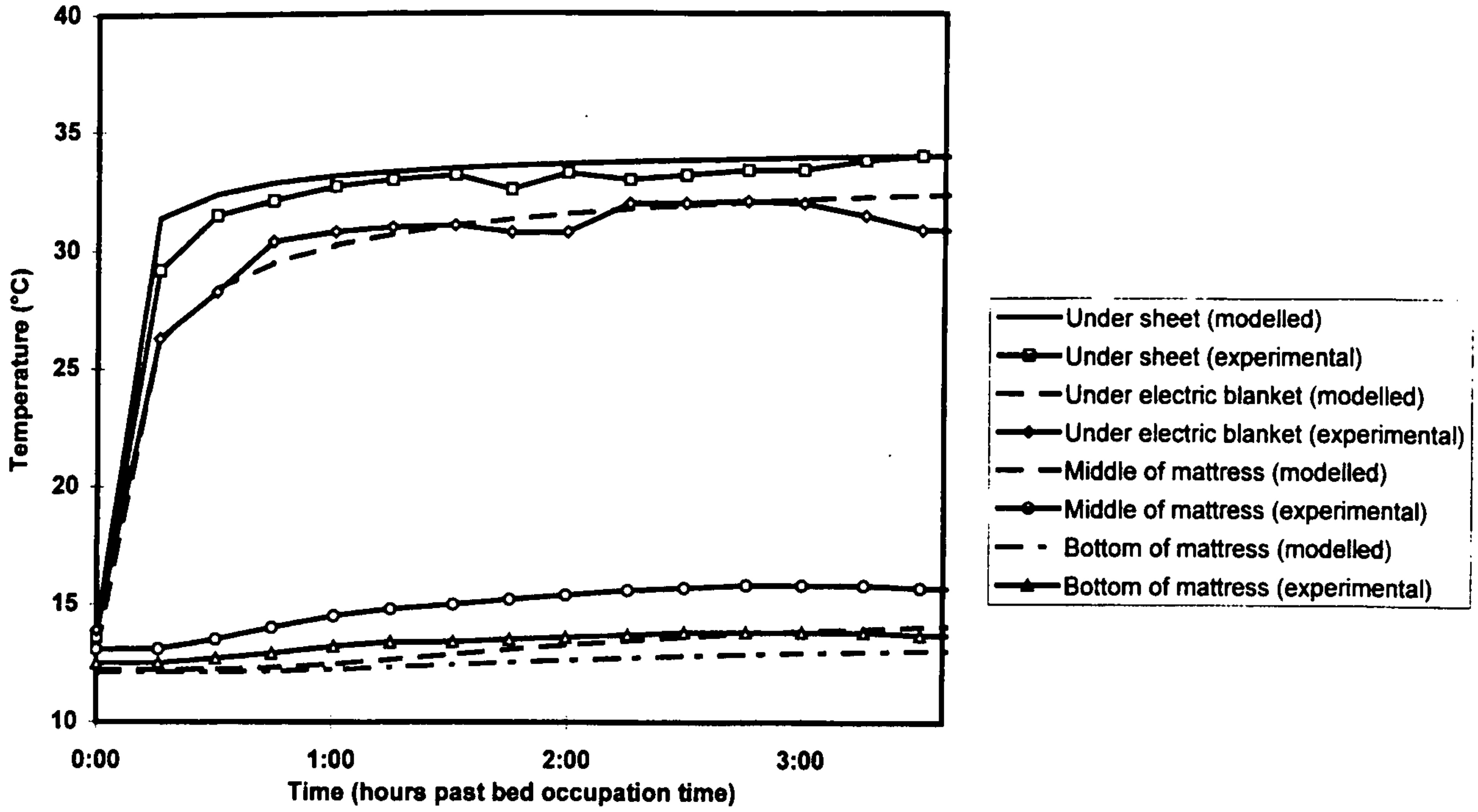


Fig. 5. Comparison between experimental and modelled relative humidities, temperatures and vapour pressures in a mattress when the bed is occupied.



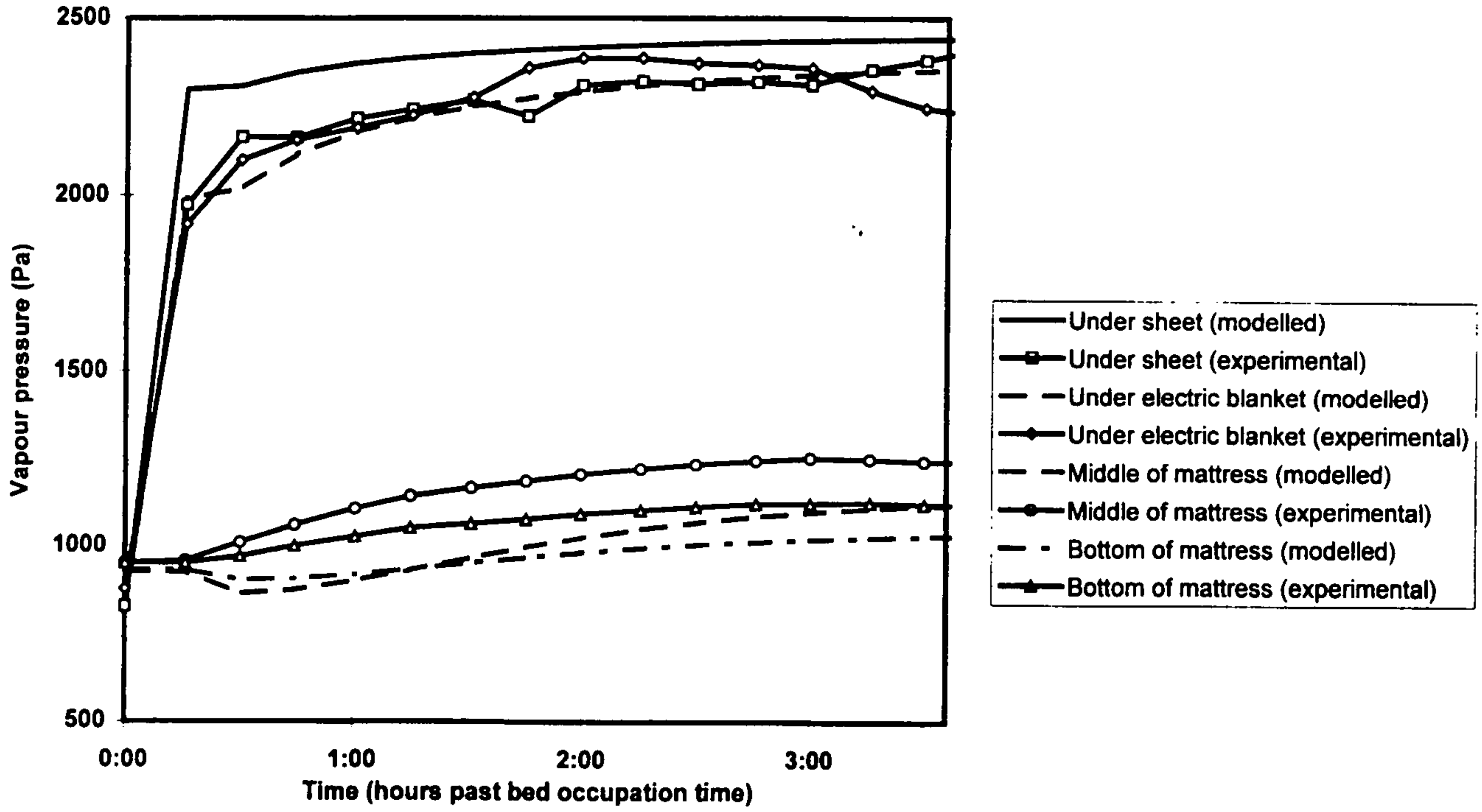


Fig. 5—*continued.*

the subfloor medium could be air if over a crawl space, or soil if the flooring is a slab on ground.

Room psychrometric boundary conditions are taken as a smoothed version of those found in uncontrolled rooms, i.e. a mean value plus a 24-h approximately sine fluctuation about that value. The standard technique of representing sinusoids as imaginary parts of complex valued exponentials and solving the differential equation in that domain will be used. Therefore the room temperature is specified as:

$$T_{room} = \bar{T}_{room} + \Delta T_{room} \sin(\omega t) = \bar{T}_{room} + \Delta T_{room} \Im(\exp(j\omega t)) \quad (18)$$

where T_{room} is the room temperature, $\bar{T}_{room}$ is the mean room temperature, ΔT_{room} is the amplitude of the 24-h component of the room temperature, ω is the angular frequency corresponding to a 24-h period (7.272×10^{-5} radians/s), $\Im$ is the imaginary part of the enclosed expression. Likewise, for vapour pressure we will have

$$p_{\text{room}} = \bar{p}_{\text{room}} + \Delta p_{\text{room}} \Im \exp(j\omega t) \tag{19}$$

The room conditions couple into the carpet through surface heat and mass transfer coefficients, i.e.

$$h(\Delta p_{\text{room}} - \Delta p_{\text{top carpet}}) = D_{\text{carpet}} \left.\frac{\partial p}{\partial x}\right|_{\text{top carpet}} \tag{20}$$

and

$$H(\Delta T_{\text{room}} - \Delta T_{\text{top carpet}}) = k_{\text{carpet}} \left.\frac{\partial T}{\partial x}\right|_{\text{top carpet}} \tag{21}$$

where h and H are the coefficients of vapour and heat transfer, respectively, and D_{carpet} and k_{carpet} are the vapour coefficient and conductivity of the carpet, respectively.

Equation (20) at the carpet surface becomes

$$h(\Delta p_{\text{room}} - \Delta p_{\text{top carpet}}) = D_{\text{carpet}} \left.\left(\frac{\partial p}{\partial \Phi}\frac{\partial \Phi}{\partial x} + \frac{\partial p}{\partial T}\frac{\partial T}{\partial x}\right)\right|_{\text{top carpet}} \tag{22}$$

where Φ is relative humidity. Temperature and vapour pressure on the bottom of the flooring layer are taken as constant.

Fluxes are matched at the carpet-floor interface giving

$$D_{\text{carpet}} \left.\frac{\partial p}{\partial x}\right|_{\text{base carpet}} = D_{\text{floor}} \left.\frac{\partial p}{\partial x}\right|_{\text{top floor}} \tag{23}$$

i.e.

$$D_{\text{carpet}} \left.\left(\frac{\partial p}{\partial \Phi}\frac{\partial \Phi}{\partial x} + \frac{\partial p}{\partial T}\frac{\partial T}{\partial x}\right)\right|_{\text{base carpet}} = D_{\text{floor}} \left.\left(\frac{\partial p}{\partial \Phi}\frac{\partial \Phi}{\partial x} + \frac{\partial p}{\partial T}\frac{\partial T}{\partial x}\right)\right|_{\text{top floor}} \tag{24}$$

and for temperature

$$k_{\text{carpet}} \left.\frac{\partial T}{\partial x}\right|_{\text{base carpet}} = k_{\text{floor}} \left.\frac{\partial T}{\partial x}\right|_{\text{top floor}} \tag{25}$$

Throughout this work conditions at the base of the carpet and the top of the floor are taken as identical, i.e. there are no contact resistances between the carpet and the floor.

Heat transfer is governed by the energy conservation equation

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c}\frac{\partial^2 T}{\partial x^2} = \alpha \frac{\partial^2 T}{\partial x^2} \tag{26}$$

where the diffusivity α is given by

$$\alpha = \frac{k}{\rho c} \tag{27}$$

using the customary notation; however, an important assumption is made that, for both the carpet and the floor,

$$d_k = \sqrt{\frac{k}{2\omega\rho c}} \gg l \tag{28}$$

where l is the thickness of the layer under consideration. This implies that the heat storage is small enough that, at all times, a linear temperature gradient is maintained in the carpet and the floor. Therefore, in addition to the gradient given by the mean temperatures $\bar{T}$, the amplitudes of the temperatures remain in phase and form linear gradients through the carpet and floor layers, i.e.

$$\frac{\partial \Delta T}{\partial x} = \frac{\Delta T_{\text{top floor}} - \Delta T_{\text{top carpet}}}{l_{\text{carpet}}} \tag{29}$$

in the carpet and

$$\frac{\partial \Delta T}{\partial x} = \frac{\Delta T_{\text{under floor}} - \Delta T_{\text{top floor}}}{l_{\text{floor}}} \tag{30}$$

in the floor, where $\Delta T_{\text{top carpet}}$ is the amplitude of the temperature fluctuation at the surface of the carpet, $\Delta T_{\text{top floor}}$ is the amplitude of the temperature fluctuation at the surface of the floor, $\Delta T_{\text{under floor}}$ is the amplitude of the temperature fluctuation at the bottom surface of the floor.

The quantity d_k is sometimes known as the apparent depth (for heat transfer) [24]. There are of course two heat transfer apparent depths of this model, d_{kc}, for the carpet and d_{kf} for the floor. Carpets and floors will have d_k in the order of centimetres which is much greater than the carpet thickness as we are assuming, but less so for the floor. However, since d_{kf} is of the same order as the floor thickness, the departure from the assumption of a linear temperature gradient in the floor will not be large enough to significantly affect the accuracy of the model being examined here.

For moisture transfer this linearising simplifying assumption is not made. Indeed, for the floor, it is assumed that the apparent depth for moisture transfer is very much less than the floor thickness, i.e.

$$d_{Df} = \sqrt{\frac{D_{\text{floor}}}{2\omega}} \ll l_{\text{floor}} \tag{31}$$

Since d_{Df} is of the order of millimetres [24] for daily periods and l_{floor} is of the order of centimetres, this inequality is well met.

The differential equation governing moisture transfer is taken as

$$\frac{\partial m}{\partial t} = D\frac{\partial^2 p}{\partial x^2} \tag{32}$$

Solutions for the mean values for temperature and vapour pressure are given by the standard solutions for constant heat and moisture flow, in particular

$$\bar{T}_{\text{top floor}} = \bar{T}_{\text{base carpet}} = \frac{\bar{T}_{\text{room}}(l_{\text{floor}}/k_{\text{floor}}) + \bar{T}_{\text{under floor}}(l_{\text{carpet}}/k_{\text{carpet}} + 1/H)}{l_{\text{carpet}}/k_{\text{carpet}} + l_{\text{floor}}/k_{\text{floor}} + 1/H} \tag{33}$$

and

$$\bar{T}_{\text{top carpet}} = \frac{\bar{T}_{\text{room}}(l_{\text{carpet}}/k_{\text{carpet}} + l_{\text{floor}}/k_{\text{floor}}) + \bar{T}_{\text{outside}}/H}{l_{\text{carpet}}/k_{\text{carpet}} + l_{\text{floor}}/k_{\text{floor}} + 1/H} \tag{34}$$

and likewise for mean vapour pressures.

Mean relative humidities are calculated from the mean temperature and vapour pressure using standard psychrometric formulae, see for instance the CIBSE handbook [23].

It is assumed that the sorption curves for the carpet and floor are linear and independent of temperature, i.e. $\partial m/\partial \Phi$ is constant. Equation (32) becomes

$$\frac{\partial m}{\partial \Phi}\frac{\partial \Phi}{\partial t} = D\left(\frac{\partial p}{\partial \Phi}\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial p}{\partial T}\frac{\partial^2 T}{\partial x^2}\right) \tag{35}$$

where $\partial p/\partial \Phi$ and $\partial p/\partial T$ describe the psychrometric relationships between vapour pressure, temperature, and relative humidity. These relations are taken as constant at their mean values over the temperatures and humidities of interest.

Since temperature gradients are being taken as approximately linear, then $(\partial^2 T/\partial x^2) \approx 0$. It is shown in the appendix that eqn (35) then becomes

$$\frac{\partial \Phi}{\partial t} = D\frac{\partial p}{\partial \Phi}\frac{\partial^2 \Phi}{\partial x^2}\Big/\frac{\partial m}{\partial \Phi} = \beta\frac{\partial^2 \Phi}{\partial x^2} \tag{36}$$

where

$$\beta = D\frac{\partial p}{\partial \Phi}\Big/\frac{\partial m}{\partial \Phi} \tag{37}$$

β can be defined as the humidity diffusivity.

Again since temperature gradients are being taken as approximately linear, then temperature amplitudes, as well as mean temperatures, are linear, i.e.

$$\Delta T_{\text{top floor}} = \Delta T_{\text{base carpet}} = \frac{\Delta T_{\text{room}}(l_{\text{floor}}/k_{\text{floor}}) + \Delta T_{\text{under floor}}(l_{\text{carpet}}/k_{\text{carpet}} + 1/H)}{l_{\text{floor}}/k_{\text{floor}} + l_{\text{carpet}}/k_{\text{carpet}} + 1/H} \tag{38}$$

and

$$\Delta T_{\text{top carpet}} = \frac{\Delta T_{\text{room}}(l_{\text{carpet}}/k_{\text{carpet}} + l_{\text{floor}}/k_{\text{floor}}) + \Delta T_{\text{under floor}}/H}{l_{\text{carpet}}/k_{\text{carpet}} + l_{\text{floor}}/k_{\text{floor}} + 1/H} \tag{39}$$

The solution to eqn (36) meeting the boundary conditions above is of the form

$$\Delta\Phi = A\sinh((1+j)x/d_{c\Phi}) + B\cosh((1+j)x/d_{c\Phi}) \tag{40}$$

multiplied by the time-dependent term $\exp(j\omega t)$, where A and B are constants yet to be defined and

$$d_\Phi = 2d_{\text{eff}} = \sqrt{\frac{2\beta}{\omega}} \tag{41}$$

where d_{eff} is the effective penetration depth [24]. $d_{c\Phi}$ will apply to the carpet and $d_{f\Phi}$ will apply to the floor.

For the range of values for β encountered in this carpet/flooring situation, and with a daily period of the driving potentials, d_Φ is of the size of millimetres [24]. The scale of a carpet pile is millimetres, but the scale of flooring is centimetres, which will cause eqn (40) to degenerate to an exponential in the flooring immediately below the carpet, i.e.

$$\Delta\Phi = \Delta\Phi_{\text{top floor}}\exp(-(1+j)x/d_{f\Phi}) \tag{42}$$

At the top of the carpet pile, considering now only the fluctuating component, eqn (40) we get

$$\left.\frac{\partial\Delta\Phi}{\partial x}\right|_{\text{top carpet}} = \frac{A(1+j)}{d_{c\Phi}} \tag{43}$$

and at the base of the carpet pile,

$$\left.\frac{\partial\Delta\Phi}{\partial x}\right|_{\text{base carpet}} = \frac{A(1+j)}{d_{c\Phi}}\cosh\left(\frac{(1+j)l_{\text{carpet}}}{d_{c\Phi}}\right) + \frac{B(1+j)}{d_{c\Phi}}\sinh\left(\frac{(1+j)l_{\text{carpet}}}{d_{c\Phi}}\right) \tag{44}$$

and at the top of the floor

$$\left.\frac{\partial\Delta\Phi}{\partial x}\right|_{\text{top floor}} = -(1+j)\frac{\Delta\Phi_{\text{top floor}}}{d_{f\Phi}} \tag{45}$$

The amplitude of vapour pressure is given by

$$\Delta p \approx \frac{\partial p}{\partial \Phi}\Delta\Phi + \frac{\partial p}{\partial T}\Delta T \tag{46}$$

Coupling eqns (42) and (46) with the boundary conditions (23) and (24) gives

$$h\left(\Delta p_{\text{room}} - \frac{\partial p}{\partial \Phi}B - \frac{\partial p}{\partial T}\Delta T_{\text{top carpet}}\right)$$
$$= D_{\text{carpet}}\left(-\frac{\partial p}{d_{c\Phi}}\frac{A(1+j)}{d_{c\Phi}} + \frac{\partial p}{dT}\frac{\Delta T_{\text{top carpet}} - \Delta T_{\text{base carpet}}}{l_{\text{carpet}}}\right) \quad (47)$$

and

$$-D_{\text{carpet}}\begin{bmatrix}\frac{\partial p}{\partial \Phi}(1+j)(A\cosh((1+j)l_{\text{carpet}}/d_{c\Phi})/d_{c\Phi} \\ +B\sinh((1+j)l_{\text{carpet}}/d_{c\Phi}))/d_{c\Phi} \\ -\frac{\partial p}{\partial T}\frac{\Delta T_{\text{top carpet}} - \Delta T_{\text{base carpet}}}{l_{\text{carpet}}}\end{bmatrix}$$
$$= D_{\text{floor}}\left(\frac{\partial p}{\partial \Phi}\frac{(1+j)\Delta\Phi_{\text{top floor}}}{d_{f\Phi}} + \frac{\partial p}{\partial T}\frac{\Delta T_{\text{top floor}} - \Delta T_{\text{under floor}}}{l_{\text{floor}}}\right) \quad (48)$$

and solving eqns (44), (47) and (48) for A, B and $\Delta\Phi_{\text{base carpet}}$ yields for $\Delta\Phi_{\text{base carpet}}$, after simplification

$$\Delta\Phi_{\text{base carpet}} = \frac{\Delta\mathbb{T}\{\mathbb{B} - 1 + (\mathbb{H}\sinh\mathbb{L} + \cosh\mathbb{L})(1 - \mathbb{D}/\mathbb{K})\} + \mathbb{H}\Delta\Phi}{(\mathbb{H}+\mathbb{D})\cosh\mathbb{L} + (1+\mathbb{H})\sqrt{\mathbb{D}}\sinh\mathbb{L}} \quad (49)$$

where all quantities are dimensionless and are defined as follows:

$$\Delta\mathbb{T} = (1-j)\frac{d_{f\Phi}}{\sqrt{2}}\left(\frac{\partial p}{\partial T}\bigg/\frac{\partial p}{\partial \Phi}\right)\left(\frac{\Delta T_{\text{top carpet}} - \Delta T_{\text{base carpet}}}{l_{\text{carpet}}}\right) \quad (50)$$

or

$$\Delta\mathbb{T} = (1-j)\frac{d_{f\Phi}}{\sqrt{2}}\left(\frac{\partial p}{\partial T}\bigg/\frac{\partial p}{\partial \Phi}\right) \times \left(\frac{\Delta T_{\text{room}} - \Delta T_{\text{under floor}}}{k_{\text{carpet}}(1/H + l_{\text{carpet}}/k_{\text{carpet}} + l_{\text{floor}}/k_{\text{floor}})}\right) \quad (51)$$

using eqns (38) and (39).

$$\mathbb{H} = \frac{(1-j)}{\sqrt{2}}\frac{hd_{f\Phi}}{D_{\text{carpet}}} \quad (52)$$

$$\mathbb{L} = (1+j)l_{\text{carpet}}/d_{c\Phi} \quad (53)$$

$$\mathbb{B} = \frac{hk_{\text{carpet}}}{HD_{\text{carpet}}} \quad (54)$$

$$\Delta\Phi = \Delta p_{\text{room}}\bigg/\frac{\partial p}{\partial \Phi} \quad (55)$$

$$\mathbb{D} = \frac{D_{\text{floor}}}{D_{\text{carpet}}} \quad (56)$$

$$\mathbb{K} = \frac{k_{\text{floor}}}{K_{\text{carpet}}} \quad (57)$$

The physical result for $\Delta\Phi_{\text{carpet}}$ is found by taking the real part of eqn (49).

The temperature at the base of the carpet is given by adding eqns (33) and (38) and the vapour pressure is found using standard psychrometric formulae [23].

Note that $\Delta\Phi$ here, eqn (55), is not the amplitude of the room relative humidity, $\Delta\Phi_{\text{room}}$. That is given as follows.

$$\Delta p_{\text{room}} \approx \frac{\partial p}{\partial \phi}\Delta\Phi_{\text{room}} + \frac{\partial p}{\partial T}\Delta T_{\text{room}} \quad (58)$$

i.e.

$$\Delta\Phi_{\text{room}} = \frac{\Delta p_{\text{room}} - \frac{\partial p}{\partial T}\Delta T_{\text{room}}}{\frac{\partial p}{\partial \phi}} = \Delta\Phi - \left(\frac{\partial p}{\partial T}\bigg/\frac{\partial p}{\partial \phi}\right)\Delta T_{\text{room}}$$

Note also that the base-of-carpet relative humidity amplitude $\Delta\Phi_{\text{base carpet}}$ does not depend on the under-floor humidity directly in eqn (49). However, indirectly, the base-of-carpet relative humidity amplitude will be influenced a little by outdoor conditions, which also influences the subfloor values. The mean value of the base-of-carpet relative humidity does depend on the under-floor humidity as this is derived from eqn (33) and its vapour pressure equivalent.

5.2. Special cases

If the surface coefficients are infinite, i.e. the conditions on the surface of the carpet are identical with those of the room air, then eqn (49) simplifies to

$$\Delta\Phi_{\text{base carpet}} = \frac{\Delta\mathbb{T}(1 - \mathbb{D}/\mathbb{K})\sinh\mathbb{L} + \Delta\Phi}{\cosh\mathbb{L} + \sqrt{\mathbb{D}}\sinh\mathbb{L}} \quad (59)$$

If, further, indoor to outdoor temperature gradients are small, eqn (49) simplifies to

$$\Delta\Phi_{\text{base carpet}} = \frac{\Delta\Phi}{\cosh\mathbb{L} + \sqrt{\mathbb{D}}\sinh\mathbb{L}} \quad (60)$$

6. Comparison with experimental results

Figure 6 shows a comparison between experimental and the calculated amplitudes of the relative humidities in the base of a carpet, using eqn (49). The comparison is illustrative only since, as in the case of bedding, actual physical values were not used; rather, a fit was made using values in the range to be expected. Values used are listed in Table 2.

The experimental data illustrated is the same as that appearing in Fig. 2, except that the average relative humidity value has been subtracted from each humidity

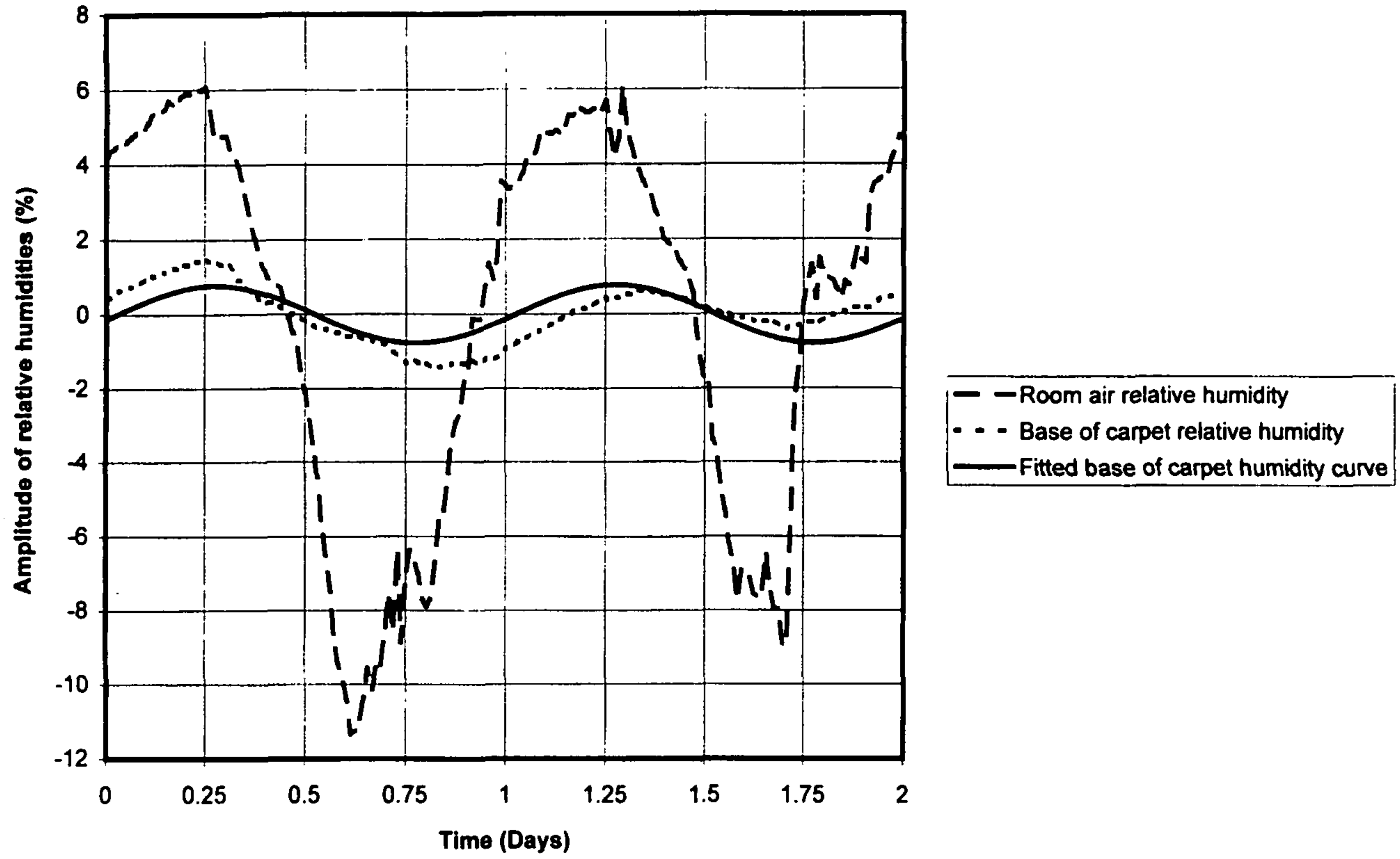


Fig. 6. Comparison between experimental and modelled relative humidities in the base of a carpet.

Table 2
Values of dimensionless quantities used to fit eqn (49) to the experimental data of carpet relative humidities

Dimensionless quantity	Value	Cross reference (equation)
$\Delta\mathbb{T}$	$0.0373(1-j)$	(50)
$\Delta\Phi$	0.169	(55)
$\mathbb{L}$	1.56	(53)
$\mathbb{B}$	$0.218(1+j)$	(54)
$\mathbb{H}$	$0.0897(1-j)$	(52)
$\mathbb{D}$	4.71	(56)
$\mathbb{K}$	3.13	(57)

to leave only the amplitude of the fluctuating components. This is equivalent to using eqn (33) and its vapour pressure equivalent to calculate the mean carpet base humidity. The modelled result reproduces the phase shift and amplitude damping of the base-of-carpet humidity. Given the degree of simplification used in deriving the analytical model, and given that the model considers only first harmonic terms, the agreement between the model and the experimental data is good.

7. Conclusions

The recent development of a very small relative humidity sensor has made it possible to measure and distinguish microclimates in microenvironments separated by only millimetres, e.g. microclimates in the base and the top of a carpet, or microclimates above and below a sheet in a bed. These microenvironments are important microhabitats for biocontaminants such as dust mites and mould. Experimental data obtained using this sensor can be quite complex and non-intuitive. Nevertheless, despite some significant simplifications, it has been possible to reproduce this, in some cases, complex behaviour using analytical models describing the heat and moisture transfer behaviour in these microenvironments. Appropriate dimensionless parameters have been identified, which should retain their significance even when accurate numerical models are used to describe these situations, and, since they describe how the system scales, may suggest different ways of achieving the same improvement in the systems' microclimates.

Appendix: Demonstration that $\partial^2 T/\partial x^2$ can be ignored in deriving psychrometric conditions in carpets

A key assumption in the development above is that at all times there is a linear temperature gradient in the carpet and the floor, see eqns (29) and (30). This in turn implies that

$$\frac{\partial^2 \boldsymbol{T}}{\partial \boldsymbol{x}^2} \approx 0 \tag{61}$$

For the differential equation conserving mass we have eqn (35), viz

$$\frac{\partial m}{\partial \Phi}\frac{\partial \Phi}{\partial t} = D\left(\frac{\partial p}{\partial \Phi}\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial p}{\partial T}\frac{\partial^2 T}{\partial x^2}\right) \tag{62}$$

It does not follow automatically that, since the second space derivative of temperature is approximately zero, eqn (61), that

$$\frac{\partial p}{\partial T}\frac{\partial^2 T}{\partial x^2} \ll \frac{\partial p}{\partial \Phi}\frac{\partial^2 \Phi}{\partial x^2} \tag{63}$$

To show this, eqn (62) needs manipulating into a form allowing the order of magnitudes of its terms to be examined.

Within the carpet or the floor the fluctuating part of the temperature solution is given from eqns (29) and (30). Including the time component of the solution gives a temperature at x of $\Delta T(x)\,e^{j\omega t}$, implying

$$\frac{\partial T}{\partial t} = j\omega \Delta T e^{j\omega t} = \alpha \frac{\partial^2 T}{\partial x^2} \tag{64}$$

At least approximately we know that the relative humidity amplitude $\Delta\phi$ is given by eqn (40), viz

$$\Delta\Phi = A\sinh((1+j)x/d_\Phi) + B\cosh((1+j)x/d_\Phi)$$

which means that

$$\frac{\partial^2 \Phi}{\partial x^2} = 2j\frac{\Delta\Phi}{d_\Phi^2} \tag{65}$$

Substituting eqns (40) and (64) into eqn (62), and dropping the time dependent term, yields

$$\frac{\partial m}{\partial \Phi}\frac{\partial \Phi}{\partial t} = jD\left(2\frac{\partial p}{\partial \Phi}\frac{\Delta\Phi}{d_\Phi^2} + \omega\Delta T\frac{\partial p}{\partial T}\right) \tag{66}$$

It can be seen for the inequality $(\partial p/\partial T)(\partial^2 T/\partial x^2) \ll (\partial p/\partial \Phi)(\partial^2 \Phi/\partial x^2)$, eqn (63), to be true requires

$$2\frac{\partial p}{\partial \Phi}\frac{\Delta\Phi}{d_\Phi^2} \gg \omega\Delta T\frac{\partial p}{\partial T} \tag{67}$$

or

$$R = \frac{\omega\Delta T d_\Phi^2}{2\Delta\Phi}\left(\frac{\partial p}{\partial T}\bigg/\frac{\partial p}{\partial \Phi}\right) \ll 1 \tag{68}$$

Using values for temperature, vapour pressure, and humidity encountered in building physics, and with a period of one day, gives R a value of the order 10^{-9} or 10^{-10}, which is small as required.

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