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A New Analytical Approach to the Long Term Behaviour of Moisture Concentrations in Building Cavities—I. Non-condensing Cavity

M. J. CUNNINGHAM*

This paper, the first of two, presents a conceptual model of moisture concentrations in a building cavity. The model is comprehensive and general considering air infiltration, vapour diffusion and material hygroscopicity under non-steady state conditions. The resulting linearised coupled differential equations are analytically solved to study the case of long term cavity moisture behaviour. Dimensionless parameters and algebraic formulae are presented describing all important moisture performance parameters for a non-condensing cavity. Two primary time constants are identified, and a third, which governs the drying rate, derived. This allows the identification of three drying regimes based on cavity tightness. Some general design recommendations are given.

NOMENCLATURE

A_i effective surface area for the flux of vapour to and from region i (m^2)
A_m effective surface area of hygroscopic material for the flux of vapour to and from the cavity (m^2)
$\bar{c}$ a weighted mean vapour concentration (kg m^{-3})
c_f final vapour concentration in the cavity (kg m^{-3})
c_i vapour concentration in region i (kg m^{-3})
c_m vapour concentration in hygroscopic material (kg m^{-3})
c_0 vapour concentration in the cavity (kg m^{-3})
C_0 initial cavity vapour concentration (kg m^{-3})
$\mathscr{c}_f$ dimensionless form of c_f
$\mathscr{c}_o$ dimensionless cavity moisture concentration parameter
F_{io} air change rate from region i to the cavity (s^{-1})
F_{oi} air change rate from the cavity to region i (s^{-1})
h lumped vapour conductivity of the hygroscopic material (kg N^{-1} s^{-1})
k proportionality constant, linearising the sorption curve of the hygroscopic material (m^2 s^{-2})
$\mathscr{K}$ dimensionless form of k
m moisture concentration in the hygroscopic material (kg m^{-3})
m_f final moisture concentration of the hygroscopic material (kg m^{-3})
m_o equilibrium moisture concentration of the hygroscopic material associated with water vapour partial pressure p_o (kg m^{-3})
M_o original hygroscopic material moisture concentration (kg m^{-3})
$\mathscr{m}$ dimensionless hygroscopic material moisture concentration parameter
p_i water vapour partial pressure in region i (N m^{-2})
p_m water vapour partial pressure in hygroscopic material (N m^{-2})
p_o water vapour partial pressure in the cavity (N m^{-2})
r_i vapour resistance from the cavity to region i (N s kg^{-1})
R universal gas constant (8310 J K^{-1} $kmole^{-1}$)
t time(s)
t_m time constant associated with hygroscopic material performance(s)
t_o time constant associated with cavity performance(s)
t_1, t_2 time constant(s)
T Kelvin temperature (K)
$\bar{T}$ mean time and space Kelvin temperature (K)
V_o volume of the cavity (m^3)
V_m volume of the hygroscopic storage medium (m^3)
W molecular weight of water (18 kg $kmole^{-1}$)
μ ratio of the surface area of the hygroscopic material to the surface area of the cavity
ν ratio of the volume of the hygroscopic material to the volume of the cavity
τ dimensionless time

*Components Division, Building Research Association of New Zealand, BRANZ, Private Bag, Porirua, New Zealand.

1. INTRODUCTION

IN NEW Zealand, as in other countries, there are two distinct but sometimes simultaneous cavity moisture problems. Firstly, there is the problem of unacceptable moisture present throughout the structure as the newly enclosed timber framing, often very wet, dries—sometimes over a period of several years. Secondly, and more seriously, certain cavities, particularly roofs, in certain localities can sometimes accumulate more moisture each winter than dries out in the following summer. This can eventually lead to serious moisture problems, occupant discomfort, and possibly even irreversible structural damage.

Remedial measures for these problems are based on experience and judgement. It is necessary to locate the sources of moisture and identify the mechanisms that allow it access to the cavity. Having established these moisture sources and their associated transport mechanisms, appropriate remedial measures can be devised. There is often, however, hardly a qualitative understanding of the drying curves to be expected from the various remedial measures applied, let alone a quantitative method that would allow a designer to predict the results of his actions.

This is partly because the interaction of the physical processes involved is not thoroughly understood, even though the processes themselves are known and quantifiable. Further, the issue is complicated by the perpetuation of two faults in some texts [1, 2], and hence into the training of designers, viz:

(1) The very important mechanism of moisture transfer by airborne convection is often ignored.

(2) The fact that cavity systems often have times to equilibrium in the order of months or years, while constantly acknowledged, is not built into conventional calculational tools. Indeed, the dew point profile method assumes a steady state condition and is not applicable until this steady state has been reached.

Providing a more correct and useful design tool requires a three part programme:

(a) The identification of the physical processes involved.
(b) The description of their interactions.
(c) The calculation of the consequences.

(1) The fundamental physical processes are in fact known. They are: firstly, water vapour diffusion and airborne convection (hereafter called leakage) into and out of the cavity from a variety of sources (e.g. outside, living space, ventilation and extraction fans, other building cavities, etc); and secondly, the probable presence in the cavity of a hygroscopic storage medium (e.g. timber, concrete, macerated paper, etc). This storage medium could be the cavity linings themselves.

(2) The description of the process interactions is performed by conserving moisture, energy and perhaps air in the cavity, and in other locations of interest, resulting in a coupled set of partial differential equations.

(3) The calculation of consequences requires the solving, by some technique, of the differential equations written down in Part 2.

Studies that have followed this three part programme are normally deficient in one or both of Parts 1 and 2. The dew point profile technique, Keiper's method [3] and Glaser's work [4] ignore the leakage mechanism. This is a serious defect, as shown by Trethowen [5], who modified the Keiper method to include leakage. Furthermore, the effect of the hygroscopic storage medium is either ignored [6], or not treated in full [7].

As regards Part 2, none of the authors mentioned have in fact chosen the differential equations approach. This is not to imply that their physics is incorrect, but merely that the powerful mathematical tools used to solve differential equations are not available to them. This means that in Part 3 of the programme, it has been necessary to resort to numerical solutions. Even where differential equations are used as the starting point, numerical methods would seem to be necessary for two reasons: firstly, the equations are complex, nonlinear and coupled, and in their unsimplified form not analytically solvable; and secondly, the cavity processes have the external climate as one of their driving forces. To give a full solution to the moisture behaviour of the cavity then, the climate must be modelled in some way that reflects its statistics, i.e. it must include such parameters as mean summer and winter temperatures and relative humidities, as well as seasonal and diurnal fluctuations of these parameters. Clearly, to do this in full requires numerical modelling.

Certain techniques, e.g. the Keiper method [3], sidestep these problems by guessing an average winter length and associated mean climate conditions in order to give an estimate of winter moisture accumulation. This technique, however, is based on judgement and experience with the location in question, and hence does not have a predictive capacity for new locations, except with further judgement. Unless a model or calculational technique can predict winter moisture accumulation, it is vulnerable to the criticism that it is of explanatory value only, or at best will only give predictions in locations already known to the designer.

These then are some of the reasons why useful analytical solutions to this problem do not yet exist. However, there is much to be gained if, by making reasonable assumptions and approximations, analytical solutions to the set of differential equations can be found.

Firstly, what analytical solutions may lack in precision they should make up for in physical insight. This insight is invaluable both for interpreting any numerical results gained with a more accurate model, and for presenting to the designer an explanation of cavity performance that physical intuition can work with.

Secondly, any experimental programme carried out to get a better understanding of the mechanisms involved and their interactions [8] will progress further and faster if a testable theory is available to direct the programme and aid in the interpretation of results.

In proceeding to analyse cavity moisture performance two cases are distinguished, the non-condensing and the condensing cavity. The latter is distinguished by the fact that during winter, for a significant period of time the cavity air temperature is below the dew point for its water vapour content so that moisture is condensed into the cavity linings and hygroscopic material. This paper analyses the non-condensing cavity and a second paper uses the results derived here as a basis for analysing the condensing cavity.

This first paper derives, from a simple conceptual model of the cavity, a set of differential equations; linearises them by making reasonable approximations and assumptions; and derives analytical solutions governing the seasonal wetting and drying in the cavity. Seasonal and diurnal sinusoidal oscillations about these drying curves are ignored in the first instance.

The solutions provide a graphic description of the seasonal behaviour and approach to equilibrium of the moisture content of the cavity air and storage medium. In detail, the performance of the building cavity is shown to be governed by two time constants: the first, called t_o in these papers, is associated with the tightness of the cavity to vapour egress both by diffusion and leakage; and the second, called t_m, is associated with the free air drying time of the timber or other storage medium. Using these and other important physical parameters, expressions for two dependent time constants—t_1 and t_2—are derived. These formulae, which give the rate of approach to equilibrium of a cavity in a non-condensing environment, are simple, comprehensive and of transparent physical meaning. They suggest the identification of three drying regimes: at one extreme, the free air drying time of the storage medium dominates and this regime is thus associated with the time constant t_m; at the other extreme, the cavity is very tight and the drying rate is thus associated with the time constant t_o; between, both time constants contribute to the rate of drying of the cavity hygroscopic material. To the author's knowledge, no previous work has clearly identified these time constants and detailed the associated drying regimes.

In addition, formulae are presented that use the above time constants and dimensionless quantities associated with the cavity geometry, to allow the designer to calculate: the time required to reach steady state; the effect of changing air leakage rates and building materials; the effect of the hygroscopicity of the storage volume, and the effect of the ratio of cavity volume to storage volume.

There is a body of experience that shows that small cavities, e.g. flat roofs of panel construction (hereafter called skillion roofs, following the New Zealand practice), are more prone to moisture accumulation problems. This paper shows that this is a consequence of the geometrical ratio, storage volume to cavity volume, called v in these papers. The formulae show a designer how to match other parameters accordingly. In particular, it is shown that to achieve the regime giving the fastest rate of drying in a smaller volume cavity, the cavity ventilation rate in air changes per hour must be increased in roughly inverse proportion to the cavity volume. On the other hand, if a cavity is drying under a tighter regime, its drying time is approximately proportional to the ratio v.

This paper concludes with some general recommendations to the designer. Specific calculational examples will appear in the second paper.

2. MODEL DEVELOPMENT AND ANALYSIS

The conceptual model of the cavity used in this paper is shown in Fig. 1. Region i represents one of n moisture sources/sinks, e.g. indoors, outdoors, other building cavities, extractor fans, etc. The two mechanisms assumed are: firstly, vapour diffusion driven by water vapour partial pressure difference $p_i - p_o$; and secondly, air convection characterised by a mean air change rate F_{io} (from region i to the cavity) and F_{oi} (from the cavity to region i) measured in s^{-1}. The cavity has volume V_o and contains a hygroscopic storage medium of volume V_m.

It is well known [9] that very wet hygroscopic materials normally dry out in two stages: (1) a constant rate stage; and (2) a falling rate stage. The first occurs when the material is in the 'funicular' state, i.e. while there are continuous threads of moisture in the pores of the material. The second stage occurs when the material is in the 'pendular' state, i.e. when there are no longer these continuous moisture threads. Once the material has dried beyond the funicular state, it is assumed that it is no longer

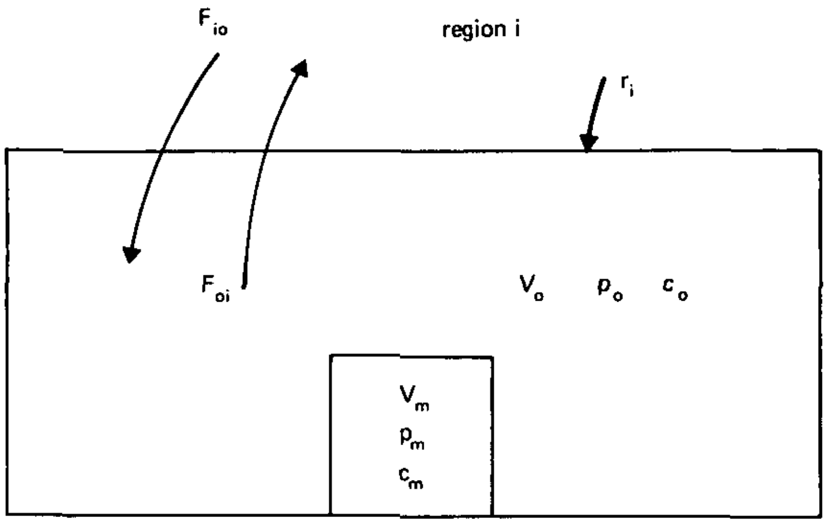

Fig. 1. Conceptual model of the building cavity.

possible to return to that state by hygroscopic absorption of moisture from the atmosphere. This means in the case of wood, for example, that the transition from the funicular state to the pendular state occurs at the wood's fibre saturation moisture concentration.

Since this paper deals only with the non-condensing situation, it is assumed that wetting and drying takes place in the pendular state, i.e. the drying exhibits a falling rate. Since this drying curve is approximately exponential, it follows that

$$\frac{dm}{dt} \propto -(m - m_0), \tag{1}$$

where m is the moisture concentration in the hygroscopic material (kg m^{-3}) and m_o is its final equilibrium value, as determined by the external water vapour partial pressure. One body of opinion [10, 11] states that moisture transfer in the pendular state, both within the material and across its boundary, is controlled by water vapour partial pressure gradients associated with the moisture content of the material in question. The vapour pressure in the hygroscopic material p_m can be calculated from the sorption curve of the material and has as its primary determining factor its moisture content; although it is also a function of temperature and the previous history of the material (hysteresis). For this work, p_m is approximated as a linear function of m and assumed to vary little with other variables. Specifically,

$$p_m = km.$$

This allows equation (1) to be written as

$$\frac{dm}{dt} \propto -(p_m - p_o),$$

where p_o is the water vapour partial pressure in the cavity, i.e.

$$\begin{aligned} V_m \frac{dm}{dt} &= -A_m h(p_m - p_o) \\ &= -A_m h(km - p_o), \end{aligned} \tag{2}$$

where h is a lumped value for vapour conductivity (kg N^{-1} s^{-1}).

It is important to note that equation (2) is approximate and lumped, modelling both surface and volume transfer of moisture to and from the hygroscopic material. This means that in measuring both internal and surface moisture movement, h is a function of at least the geometry of the material, the material type, temperature conditions, and air flow velocity across the drying surface. All these vary with time of course, but for the purposes of this paper a mean fixed value for h is taken. The h in equation (2) is not the surface transfer coefficient, but exists merely to put a parameter on the drying curve of the particular lump of material present in a given cavity under the long term mean conditions prevailing.

Conservation equations for moisture in the cavity, moisture in the hygroscopic material and air in the cavity are now presented.

Increase in cavity moisture per unit time = flux of moisture from all regions by diffusion + flux of moisture into the cavity from all regimes by leakage − flux of moisture out of the cavity to each region + flux of moisture from the hygroscopic material,

$$V_o\frac{\mathrm{d}c_o}{\mathrm{d}t}=\sum_i\left[A_i\left(\frac{p_i-p_o}{r_i}\right)+V_o(F_{io}c_i-F_{oi}c_o)\right]$$
$$+A_m h(km-p_o). \quad (3)$$

Increase of hygroscopic moisture per unit time = − loss of moisture to the cavity,

$$V_m\frac{\mathrm{d}m}{\mathrm{d}t}=-A_m h(km-p_o). \quad (2)$$

Net flow of air into the cavity = 0,

$$\sum_i(F_{io}-F_{oi})=0. \quad (4)$$

Note that the leakage term $F_{io}c_i-F_{oi}c_o$ in equation (3) assumes perfect mixing of the moist air fluxes in the cavity.

The water vapour partial pressures p_i are converted to concentrations c_i by assuming water vapour to be an ideal gas, i.e.

$$p_i=\frac{c_i RT}{W}.$$

To make progress, the complicating effect of temperature is ignored in analysing the non-condensing problem and a mean temperature $\bar{T}$, time and region independent, is used. This will not change the qualitative behaviour predicted, nor indeed have a significant effect on the long term quantitative behaviour, because the long equilibrium times involved allow daily and even yearly temperatures to be averaged. Temperature differences between regions affect vapour pressures and hence diffusion rates to a much less degree than the uncertainties in the value of the vapour resistances and air exchange rates. Temperature effects are reintroduced in the condensing case, analysed in the second paper.

Equation (3) can now be written as

$$V_o\frac{\mathrm{d}c_o}{\mathrm{d}t}=A_m h\left(km-\frac{c_o R\bar{T}}{W}\right)$$
$$+\sum_i\left[\frac{R\bar{T}}{W}\frac{A_i(c_i-c_o)}{r_i}+V_o(F_{io}c_i-F_{oi}c_o)\right] \quad (5)$$

and equation (2) as

$$V_m\frac{\mathrm{d}m}{\mathrm{d}t}=-A_m h\left(km-\frac{c_o R\bar{T}}{W}\right). \quad (6)$$

The process of putting these equations in dimensionless form begins by defining two key time constants, t_o and t_m,

$$\frac{1}{t_o}=\sum_i\left(\frac{R\bar{T}}{WV_o}\frac{A_i}{r_i}+F_{oi}\right), \quad (7)$$

$$\frac{1}{t_m}=\frac{hkA_m}{V_m}. \quad (8)$$

Physically, t_o is a measure of the rate of response of the cavity vapour concentration to vapour egress, both by diffusion and leakage; while t_m is a measure of the rate of response of the hygroscopic material's moisture content to change in external vapour pressure.

Other quantities defined are

$$\mathscr{K}=\frac{kW}{R\bar{T}} \quad (9)$$

which is a dimensionless form of k and has a value of the order of 10^{-4} for wood;

$$c_o=\frac{c_o}{C_0}\quad\text{and}\quad m=\frac{m}{M_0} \quad (10)$$

which are dimensionless forms of c_o and m respectively, where C_o and M_o are initial vapour concentrations and moisture concentrations respectively;

$$\tau=\frac{t}{t_0} \quad (11)$$

which is a dimensionless time;

$$\nu=\frac{V_m}{V_o}. \quad (12)$$

Before making the last definition, it must be re-emphasised that this work is concerned with determining the drying curves of the cavity: that is, with tracing its approach to equilibrium. The seasonal sinusoidal (approximately) fluctuations in external vapour pressure superimpose (under the linearity simplifications) sinusoidal fluctuations in the cavity moisture content. Since these are not of interest at this stage, it is necessary to use only yearly mean vapour concentrations, with seasonal, daily and other fluctuations averaged to zero. In other words, all the c_i's are constants. Fluctuating vapour concentrations are introduced in the second paper to study the condensing case.

With this understanding of constant c_i's, a weighted mean concentration $\bar{c}$ is defined as

$$\bar{c}=\frac{\sum_i[(R\bar{T}/WV_o)(A_i/r_i)+F_{io}]c_i}{\sum_i[(R\bar{T}/WV_o)(A_i/r_i)+F_{io}]}$$
$$=t_o\sum_i\left(\frac{R\bar{T}}{WV_o}\frac{A_i}{r_i}+F_{io}\right)c_i. \quad (13)$$

Using these definitions, equations (5) and (6) take on the following dimensionless form:

$$\left(\frac{\mathrm{d}}{\mathrm{d}\tau}+1+\frac{\nu}{\mathscr{K}}\frac{t_o}{t_m}\right)c_o-\frac{\nu t_o}{t_m}\frac{M_o}{C_o}m=\frac{\bar{c}}{C_o}$$
$$\left(\frac{\mathrm{d}}{\mathrm{d}\tau}+\frac{t_o}{t_m}\right)m-\frac{1}{\mathscr{K}}\frac{t_o}{t_m}\frac{C_o}{M_o}c_o=0.$$

This is a pair of linear coupled first order differential equations, whose solution with the initial conditions given is straightforward.

$$c_o=m_o=1,\ \text{when}\ \tau=0.$$

Specifically, we find

$$m=A\,\mathrm{e}^{\alpha\tau}+B\,\mathrm{e}^{\beta\tau}+\frac{\bar{c}}{M_o\mathscr{K}}$$

and

$$c_o=C\,\mathrm{e}^{\alpha\tau}+D\,\mathrm{e}^{\beta\tau}+\frac{\bar{c}}{C_o}, \quad (14)$$

where

$$A=\frac{(t_o/t_m)[(C_o/M_o\mathscr{K})-1]+\beta[(\bar{c}/M_o\mathscr{K})-1]}{\alpha-\beta},$$

$$B=\frac{(t_o/t_m)[1-(C_o/M_o\mathscr{K})]+\alpha[1-(\bar{c}/M_o\mathscr{K})]}{\alpha-\beta},$$

$$C=\frac{\mathscr{K}M_o}{C_o}\left(\frac{t_m}{t_o}\alpha+1\right)A,$$

$$D=\frac{\mathscr{K}M_o}{C_o}\left(\frac{t_m}{t_o}\beta+1\right)B, \tag{15}$$

and

$$\alpha,\beta=-\frac{1}{2}\left\{\left(\frac{t_o}{t_m}+\frac{v}{\mathscr{K}}\frac{t_o}{t_m}+1\right)\pm\sqrt{\left[\left(\frac{t_o}{t_m}+\frac{vt_o}{\mathscr{K}t_m}+1\right)^2-\frac{4t_o}{t_m}\right]}\right\}. \tag{16}$$

Note: (1) α and β are both real and negative.

(2) $$\alpha\beta=\frac{t_o}{t_m}.$$

(3) $$\alpha+\beta=-\left(\frac{t_o}{t_m}+\frac{vt_o}{\mathscr{K}t_m}+1\right). \tag{17}$$

These solutions have, as an immediate consequence, expressions for the important quantities of final equilibrium cavity moisture concentration and final hygroscopic material moisture content, since as $t\rightarrow\infty$, then $e^{\alpha\tau}$ and $e^{\beta\tau}\rightarrow 0$,

i.e. $$m\rightarrow\frac{\bar{c}}{M_o\mathscr{K}}=m_f$$

$$\therefore m_f=\frac{\bar{c}}{\mathscr{K}} \tag{18}$$

and

$$c_o\rightarrow\frac{\bar{c}}{C_o}=c_f$$

$$\therefore\ c_f=\bar{c}. \tag{19}$$

Equation (18) is merely a reflection of the fact that at equilibrium, the hygroscopic material moisture concentration must be that determined by the cavity moisture concentration through the material's sorption curve.

All other moisture performance parameters of interest, including time to equilibrium and a full time history of moisture concentrations in the process of drying or wetting from given initial conditions, are implicit in the solutions given by equations (14)–(17).

The general result derived in equation (14) illustrates the power of analytical solutions, in that it gives a complete description of the non-condensing cavity's moisture performance. Although in principle nothing else need be said, further investigation shows that formulae (14)–(16) simplify over the range of physical parameter values met in building cavities. Specifically, for wood,

$$10^{-2}\lesssim v\lesssim 10^{-1},$$

$$\mathscr{K}\sim 10^{-4},$$

$$t_o<10^5\ \text{s},$$

and

$$t_m\sim 10^6\ \text{s}.$$

Within these limits, it can be shown that equation (16) reduces to a linear expression for α,

$$\alpha\simeq\frac{vt_o}{\mathscr{K}t_m}+1$$

while since

$$\alpha\beta=\frac{t_o}{t_m}$$

then

$$\beta\simeq\frac{1}{(v/\mathscr{K})+(t_m/t_o)}.$$

These simplifications, in turn, effect simplications in the values of A, B, C and D of equation (15). Finally, we find

$$m=(M_o-m_f)e^{-t/t_2}+m_f, \tag{20}$$

where

$$t_2\simeq\frac{t_ov}{\mathscr{K}}+t_m. \tag{21}$$

Equation (20) shows that the hygroscopic material in the cavity dries exponentially under all conditions, with a time constant t_2 given by the simple linear equation (21), see Fig. 2.

The formula for the cavity vapour concentration does not simplify to the same extent. A second time constant t_1 arises, associated with α. Specifically,

$$t_1=\frac{t_o}{1+(t_ov/t_m\mathscr{K})}. \tag{22}$$

Physical implications of the drying equations

Equation (20) shows that in all cases the hygroscopic material in the cavity dries exponentially at a rate determined by t_2. In turn t_2 is given by the simple linear equation,

$$t_2=\frac{t_ov}{\mathscr{K}}+t_m. \tag{21}$$

The importance of equation (21) cannot be overemphasised. Although all parameters (with the possible exception of v) have been recognised as being important in the determination of the cavity drying rate, they have not been previously linked together in a simple, comprehensive and physically obvious fashion.

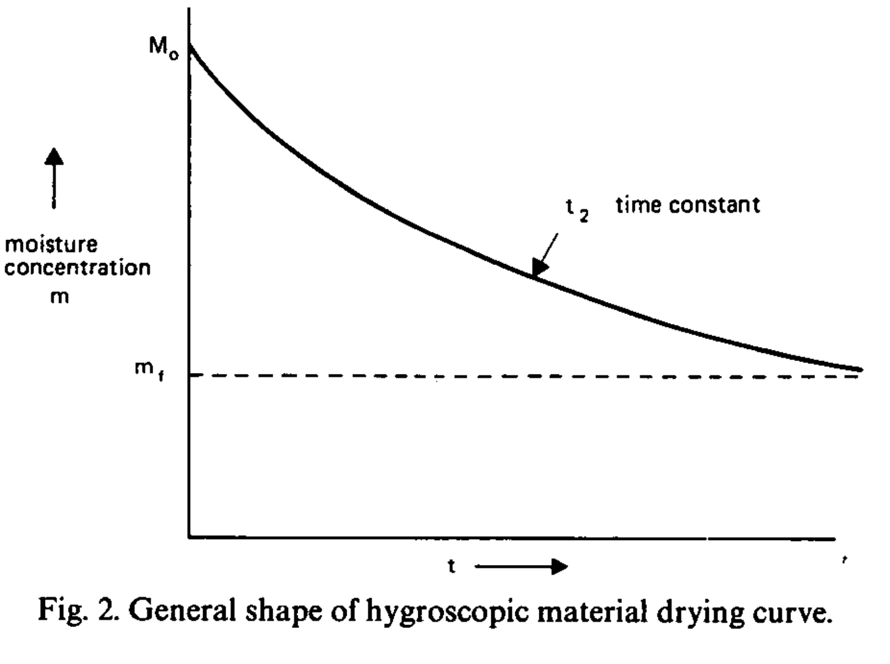

Fig. 2. General shape of hygroscopic material drying curve.

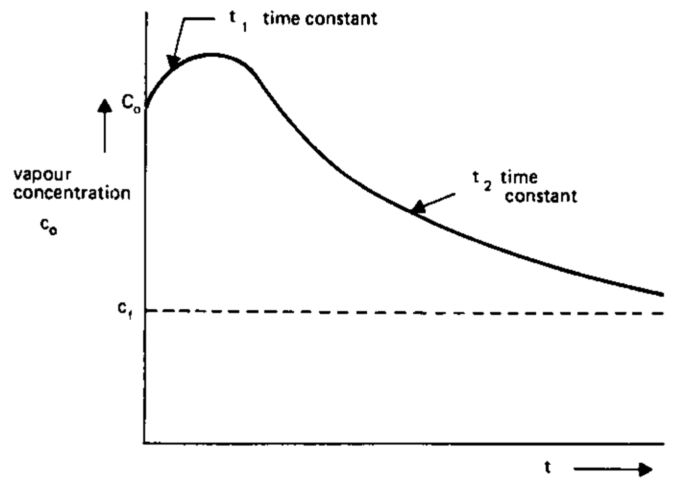

Fig. 3. General long term shape of vapour concentration curve (seasonal effects excluded).

Physically, three distinct drying regimes can be associated with time constant t_2.

(i) Hygroscopically controlled.

$$\frac{t_o v}{\mathscr{K} t_m} \ll 1, t_2 \simeq t_m.$$

(ii) Construction controlled.

$$\frac{t_o v}{\mathscr{K} t_m} \gg 1, t_2 \simeq \frac{v t_o}{\mathscr{K}}.$$

(iii) Intermediate.

$$\frac{t_o v}{\mathscr{K} t_m} \sim 1, \quad t_2 = \frac{v t_o}{\mathscr{K}} + t_m.$$

The distinction between these three regimes is shown clearly in Fig. 4, a plot of the log of the (normalised) cavity drying rate time constant t_2/t_m against the log of the (normalised) cavity tightness time constant t_o/t_m for various values of $v/\mathscr{K}$.

Further discussion of each drying regime follows.

(i) Hygroscopically controlled drying regime.

Defined as $\dfrac{v t_o}{\mathscr{K} t_m} \ll 1.$

At this limit $t_1 \simeq t_o$ and $t_2 \simeq t_m$. (23)

This is the fastest drying case and would normally be the designer's choice. Physically, this regime is associated with a cavity that can 'breathe' freely, i.e. its response time to changes in external vapour pressure is very much faster than the response time of the moisture content of the hygroscopic material.

The rate of drying of the cavity hygroscopic medium is given by equation (20), while for the vapour concentration at this limit, $D \simeq 0$ to give:

$$c_o = (C_o - c_f)\,e^{-t/t_1} + c_f, \tag{24}$$

where $t_1 \simeq t_o$.

At this limit, the differential equations are uncoupled, with the cavity and the hygroscopic material behaving as if the other did not exist.

If the hygroscopic material is wood, $\mathscr{K}$ is of the order of 10^{-4}. This drying regime is thus determined by

$$\frac{t_m}{t_o} \gg 10^4 v, \tag{25}$$

say

$$\frac{t_m}{t_o} > 10^5 v. \tag{26}$$

Clearly, cavity construction dictates whether or not this regime is achieved. For example, in the case of a roof cavity consisting of an attic space with an average height of the order of 1 m (hereafter called a pitched roof), v may perhaps be of the order of 10^{-2}. In a skillion roof, on the other hand, there will be relatively much more timber present and v may be of the order of 10^{-1}.

In these cases then, the hygroscopically controlled drying regime is achieved if

$$\frac{t_m}{t_o} > 10^3, \quad \text{pitched roof,}$$

$$\frac{t_m}{t_o} > 10^4, \quad \text{skillion roof.}$$

In either case, it can be seen that for fixed t_m, t_o must be relatively small. Achieving this fastest drying regime, from the definition of t_o (formula (7)), requires the cavity to be far

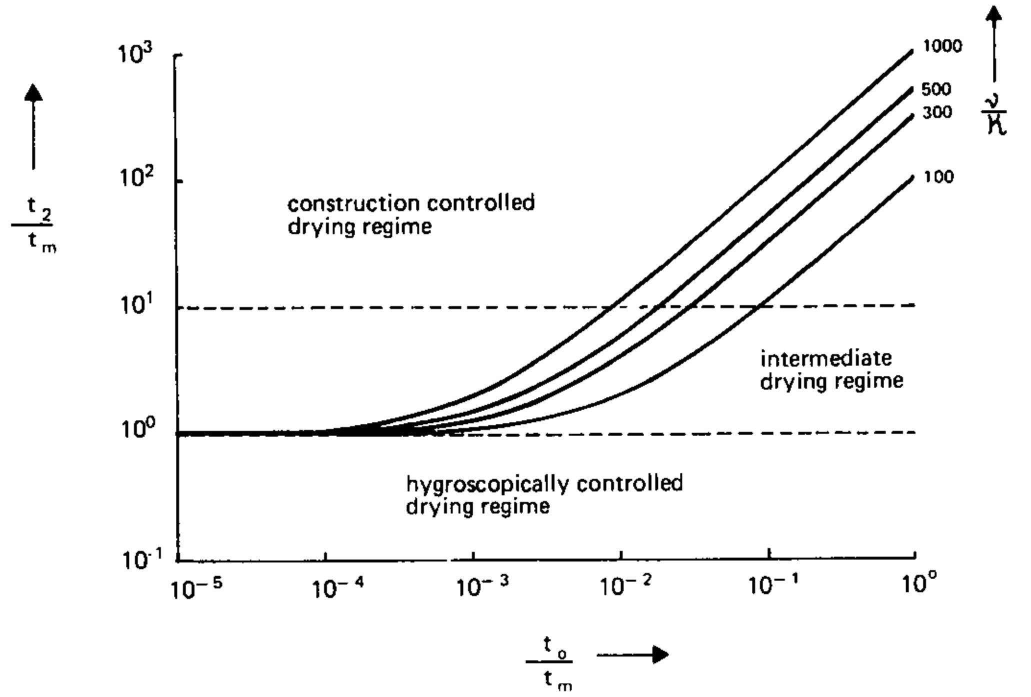

Fig. 4. Normalised plot of the drying time constant, t_2, against the cavity tightness time constant, t_o.

from tight; a state attained either by having low vapour resistance for the cavity linings and/or high leakage rates out of the cavity.

From the definitions given, the designer may in fact calculate the minimum leakage rates or maximum vapour resistance required to achieve this regime. In particular using the definition of t_o, formula (7) and using equation (26) as the requirement to be in the hygroscopically controlled regime we find

$$\sum_i F_{oi} > v\left(\frac{10^5}{t_m} - \frac{1}{V_m}\sum_i \frac{R\bar{T}A_i}{Wr_i}\right) \propto v. \tag{27}$$

In giving design limits on the smallest allowable leakage rates to achieve the hygroscopically controlled drying regime, formula (27) shows that for a fixed volume of hygroscopic medium V_m, the total air change rate required to achieve this regime is proportional to v; that is, inversely proportional to the cavity volume. This has the important consequence that a skillion roof, for example, will not achieve this hygroscopically controlled drying regime unless its air change rate is something like ten times the rate of a pitched roof.

As an example, consider a pitched roof, $v = 10^{-2}$, with linings of vapour resistance 2×10^9 N s kg^{-1} and $t_m = 1$ month $= 2.6 \times 10^6$ s. Using formula (27), we find that the total air leakage from the cavity, F_{oi}, must be greater than 4.5×10^{-4} s^{-1} (i.e. 1.6 air changes per hour) to achieve the hygroscopically controlled drying regime. If all else remains unchanged but the cavity air volume is reduced to one tenth its previous value ($v = 10^{-1}$), this drying regime can only be achieved with a total air change rate of 16 hr^{-1}.

It is of interest to note that this relationship is no longer true, if a cavity has exactly zero ventilation. A similar analysis shows that the larger t_o value required for a small volume cavity is achieved automatically in changing its volume. In other words, if an unventilated cavity achieves the hygroscopically controlled drying regime, it will continue to achieve that regime independently of any internal volume changes. Since most previous design tools seem to have ignored cavity ventilation, this may be the reason designers have not perceived cavity volume change to be an important parameter.

(ii) Construction controlled drying regime.

$$\text{Defined as} \quad \frac{vt_o}{\mathscr{K}t_m} \gg 1.$$

$$\text{At this limit} \quad t_1 \simeq \frac{\mathscr{K}t_m}{v} \quad \text{and} \quad t_2 \simeq \frac{vt_o}{\mathscr{K}}. \tag{28}$$

This is the slowest drying regime, being associated with a very tight cavity.

The cavity drying time constant t_2 is sensitive to the cladding material used, and the rates of air leakage, and is potentially very long—perhaps many years. t_2 is proportional to v, which implies that the time taken to equilibrium is inversely proportional to the cavity volume (approximately, as t_o in equation (28) does contain a term proportional to cavity volume, see equation (7)). Once again a quantitative explanation is provided for the poorer drying performance of small volume cavities.

The rate of drying for the cavity hygroscopic medium is given by equation (20), while for the vapour concentration at this limit it is found that

$$c_o = (C_o - \mathscr{K}M_o)e^{-t/t_1} + (\mathscr{K}M_o - c_f)e^{-t/t_2} + c_f. \tag{29}$$

The cavity vapour concentration rises quickly, at a rate controlled by t_1, to the relative humidity determined by the hygroscopic material's sorption curve. Thereafter, it remains in equilibrium with the hygroscopic material, as its moisture concentration falls slowly at a rate controlled by t_2, i.e. the tightness and the size of the cavity, see Fig. 3. If t_m is the order of a month, this regime would be achieved in a skillion roof by having t_o greater than say 3×10^4 s; that is, a leakage rate of less than the order of 0.1 hr^{-1} and a vapour resistance greater than say 30×10^9 N s kg^{-1}. These figures would change to 0.01 hr^{-1} and 300×10^9 N s kg^{-1} in the case of a pitched roof; illustrating the fact that it is much easier for a small volume cavity, such as a skillion roof, to achieve, inadvertently or otherwise, this slow drying regime.

(iii) Intermediate drying regime.

$$\text{Defined as} \quad \frac{t_o v}{\mathscr{K}t_m} \sim 1.$$

In this case, no simplifications are available, so

$$t_1 = \frac{t_o}{1 + (t_o v / t_m \mathscr{K})} \quad \text{and} \quad t_2 = \frac{vt_o}{\mathscr{K}} + t_m. \tag{30}$$

The rate of drying of the cavity hygroscopic medium is given by equation (20), while the cavity vapour concentration is given by equation (14), with C and D as given in equation (15). Physically, the drying rate is determined by the hygroscopic material and the cavity tightness. It is expected that most cavities normally regarded as tight will fall into this regime.

3. RECOMMENDATIONS

(1) Designers need to be much more aware of the importance of air leakage in and out of the cavity as a major contributor to its moisture performance. Air leakage often completely dominates the vapour diffusion mechanism.

(2) Previous techniques and experience make it clear that higher vapour resistance materials should be placed on the warm side of the cavity. This work extends this statement to recommend that lower air leakage is also required on the warm side of the cavity.

(3) Designers should be aware at least qualitatively of how t_2, the cavity drying rate constant, changes depending on cavity construction details, so that designs can be tailored accordingly. The hygroscopically controlled regime has the advantage of fastest drying rates, while the tighter regimes usually have the advantage of lower seasonal swings in the moisture content of the hygroscopic material.

(4) Designers should be aware of the importance of the volume ratio v; that is, the ratio of the volume of the hygroscopic material to the volume of the cavity. This ratio appears directly in the formula for the cavity drying time constant t_2 and if v is large (e.g. skillion roof type construction), the ratio has a major influence on the value of t_2.

4. CONCLUSIONS

This work has introduced from the beginning all major mechanisms involved in the moisture performance of building cavities. Consequently it has been able to give comprehensive solutions to the long term moisture performance of these cavities. In doing so, it contains many results which appear new and provide a useful insight into cavity behaviour. Amongst these are:

(i) The identification of two time constants, t_o and t_m, governing the tightness of the cavity and the drying rate of the hygroscopic medium respectively.

(ii) The derivation of simple and important formulae for the time constants t_1 and t_2, the latter determining the drying rate of the cavity hygroscopic material.

(iii) The highlighting of the importance of v, the ratio of the volume of the hygroscopic material to the volume of the cavity.

The analysis to date has been concerned with long term cavity moisture behaviour; consequently it has been sufficient to ignore annual, diurnal and other fluctuations, and deal instead with annual mean values. The second paper dealing with the condensing cavity has to consider these fluctuations. Nevertheless it draws upon the ideas outlined here, in particular the concept of cavity drying time t_2.

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