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# VALIDATING THE ISOTHERMAL PLANES METHOD FOR R-VALUE PREDICTIONS

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## ABSTRACT

*This paper describes an attempt to calibrate the Isothermal Planes method for R-value predicting, using previously published results. A set of 84 cases have been collected, for which measured R-values have been published in major publications, together with sufficient detail to allow calculation. This collection includes wood-framed, wood-plank, metal-framed, and masonry structures.*

*Comparison of R-values for this set shows good agreement between measured values and those calculated by the Isothermal Planes method (1), provided that a slightly tighter set of definitions than normal is followed. These include allowance for the small contact resistances between frame and facing (this is significant only with metal frames), as well as better rules about how layers should be defined. The R-value forecasts over the whole set were within  $\pm 0.1 \text{ m}^2\text{C/W}$  of measured values for 81% of the cases, and percentage differences were larger at smaller R-values. The average was within 2% of the measured values, indicating that there was little or no consistent bias. The standard deviations were 10% to 17% for all three material groups.*

*Forecasts of appropriate quality for engineering purposes appear to be obtainable with the minor improvements suggested here to the Isothermal Planes method, but the Parallel Flow method is not adequate and cannot be made so. Assurance of correct procedures in practice remains a problem: Continued measurements must still be used as a final arbiter.*

## INTRODUCTION

For many years there has been debate over simplified methods for R-value prediction. Several authors have argued in favor of the Isothermal Planes method, notably Valore (1980), in respect of masonry construction. This was subsequently endorsed by van Geem (1986), who demonstrated how masonry performance could be well explained by this method. Trethowen (1986) presented data showing that where Isothermal Planes and Parallel Flow methods gave significantly different answers, the Parallel Flow method gave poorer fit and, at times produced over-optimistic forecasts. Where the Isothermal Planes method was in error, the error was much smaller, and not consistently either over- or underestimated. This was taken further in 1988 (Trethowen 1988),

when it was pointed out that allowance for the small contact resistances between metal frames and their facings produced greatly improved fit in the Isothermal Planes method, to similar quality as wood framed and masonry walls. However, such inclusions have virtually zero effect in the Parallel Flow method (Trethowen 1986). Further, it has since been demonstrated (Carson et al. 1993) that not only does allowance for contact resistance give better fit in calculation, but that contact resistance is a real physical effect.

In investigating these methods, most authors seem to have concentrated on semitheoretical arguments, often concluding that, because the Isothermal Planes and the Parallel Flow methods represent the opposite extreme representations of the heat flow path distribution possibilities, the best answer must lie between the two. This appears to be the reason for CIBS (CIBS 1970) recommending the averaging of the two values. While there must be a validity that the truth must lie between the two methods, in this paper it will again be argued that the Isothermal Planes method is the more realistic in real cases.

The details of the Isothermal Planes method used in this paper are similar to those given in existing handbooks (ASHRAE 1994, CIBSE 1970), but they attempt to more specifically define how the decisions should be made, and include specific methods for metal framed structures. The development of these particular methods arose slowly over 25 years of simultaneous calculation and testing using guarded hotbox equipment in our laboratory. During this time, minor "fixes" to both Isothermal Planes and Parallel Flow methods have been tried, and those which seemed to give consistent improvement to the forecasting quality have been retained. The outcome of this evolutionary process has led to the selection of the Isothermal Planes method, which has been distilled into 11 rules, which are now suggested for more general application, perhaps with further development.

The rules are in three major groups, and are defined here in "Rules."

## THE DATA SET

A survey of literature on this subject revealed 84 cases for which measured R-values have been published, together with sufficient detail to allow proper calculation. These data are presented in Table 1. They include 15 cases of wood-frame

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**TABLE 1**  
**Table of Published Measured and Calculated R-Values**

Case No.	Source Ref.	Table entry	Calc.	Meas	Ideal Value	Meas/Calc.	Meas/Ideal	Type
1	7	5.x.2	2.9	2.4	3.2	0.83	0.75	wframe
2		5.x.5	2.3	2.3	3.2	1	0.72	wframe
3		5.x.1	2.2	2.1	3.2	0.95	0.66	wframe
4	6	4.x.1	2.3	2.2	2.6	0.96	0.85	wframe
5		4.x.2	1.8	1.8	2.6	1	0.69	wframe
6		4.x.3	1.4	1.7	2.6	1.21	0.65	wframe
7		4.x.4	1.2	1.5	2.6	1.25	0.58	wframe
8	9	1.1.1	0.37	0.37	0.35	1	1.06	wframe
8(a)			0.44	0.48	0.42	1.09	1.14	wframe
9		1.1.3	0.89	0.85	0.87	0.96	0.98	wframe
10		1.1.7	1.04	1.05	1.02	1.01	1.03	wframe
11		1.1.8	1.06	1.1	1.26	1.04	0.87	wframe
12		1.1.11	1.61	1.55	1.78	0.96	0.87	wframe
13		1.1.10	1.53	1.4	1.92	0.92	0.73	wframe
14		1.1.13	1.63	1.8	1.96	1.1	0.92	wframe
15		1.1.2	0.75	0.75	0.9	1	0.83	plank
16		1.1.5	0.92	1	1.62	1.09	0.62	plank
17		1.1.12	1.63	1.55	1.91	0.95	0.81	plank
18		1.1.6	1.06	1.03	1.3	0.97	0.79	wroof
19		1.1.14	2.14	2.3	2.4	1.07	0.96	wroof
20		1.1.15	3.23	3.1	3.53	0.96	0.88	wroof
21	9	1.2.1	0.29	0.28	0.32	0.97	0.88	block
22		1.2.2	0.25	0.27	0.32	1.08	0.84	block
23		1.2.3	0.3	0.31	0.32	1.03	0.97	block
24		1.2.4	0.31	0.31	0.32	1	0.97	block
25		1.2.5	0.54	0.5	0.64	0.93	0.78	block
26		1.2.6	0.76	0.73	2.11	0.96	0.35	block
27		1.2.7	0.5	0.45	1.37	0.9	0.33	block
28		1.2.8	0.54	0.63	2.48	1.17	0.25	block
29		1.2.9	0.51	0.49	2.48	0.96	0.2	block
30		1.2.10	0.7	0.5	2.48	0.71	0.2	block
31		1.2.11	0.64	0.6	1.18	0.94	0.51	block
32		1.2.12	0.82	0.72	1.63	0.88	0.44	block
33		1.2.13	0.69	0.73	1.18	1.06	0.62	block
34		1.2.14	0.89	1	1.87	1.12	0.53	block
35		1.2.15	0.71	0.65	1.56	0.92	0.42	block
36		1.2.16	1.1	0.77	1.56	0.7	0.49	block
37		1.2.17	1.53	1.56	1.56	1.02	1	block
38	13	5.3e.1	0.52	0.55	4.57	1.06	0.12	block
39		5.3e.2	0.46	0.48	2.53	1.04	0.19	block

**TABLE 1 (continued)**  
**Table of Published Measured and Calculated R-Values**

Case No.	Source Ref.	Table entry	Calc.	Meas	Ideal Value	Meas/Calc.	Meas/Ideal	type
40	13	5.3e.3	0.44	0.43	2.53	0.98	0.17	block
41		5.3e.4	0.44	0.43	2.53	0.98	0.17	block
42		5.3e.5	0.44	0.43	2.07	0.98	0.21	block
43		5.3e.6	0.44	0.43	3.1	0.98	0.14	block
44		5.3e.7	0.5	0.5	2.53	1	0.2	block
45		5.3e.8	0.37	0.39	2.53	1.05	0.15	block
46		5.3e.9	0.34	0.37	2.53	1.09	0.15	block
				e = empty	core			
47		5.3e.1	0.31	0.33	2.53	1.06	0.13	block
48		5.3e.1	0.31	0.49	2.53	1.58	0.19	block
49		5.3f.1	1.27	1.27	4.57	1	0.28	block
50		5.3f.2	1.03	0.93	2.53	0.9	0.37	block
51		5.3f.3	0.93	0.8	2.53	0.86	0.32	block
				f= filled	core			
52		5.3f.4	0.88	0.74	2.53	0.84	0.29	block
53		5.3f.5	0.98	0.7	2.07	0.71	0.34	block
54		5.3f.6	0.68	0.63	3.1	0.93	0.2	block
55		5.3f.7	1.47	1.18	2.53	0.8	0.47	block
56		5.3f.8	0.74	0.61	2.53	0.82	0.24	block
57		5.3f.9	0.61	0.55	2.53	0.9	0.22	block
58		5.3f.1	0.46	0.5	2.53	1.09	0.2	block
59		5.3f.1	0.5	0.8	2.53	1.6	0.32	block
60	10	1.1.4	0.87	0.9	1.61	1.03	0.56	metal
61		1.1.9	1.43	1.4	1.61	0.98	0.87	metal
62	10(ex 3)	2.1.1	1.35	1.17	2.22	0.87	0.53	metal
63	10	2.1.2a	1.23	1.2	2.17	0.98	0.55	metal
64		2.2.2b	1.67	1.85	2.34	1.11	0.79	metal
65		2.1.3a	0.97	1.1	1.61	1.13	0.68	metal
66		2.1.4a	1.3	1.3	2.56	1	0.51	metal
67		2.1.4b	1.8	1.5	2.85	0.83	0.53	metal
68		2.1.5a	1.84	1.8	2.66	0.98	0.68	metal
69	3	6.3.1	1.74	1.7	2.16	0.98	0.79	metal
70		6.3.2	1.74	1.66	2.16	0.95	0.77	metal
71		6.3.3	1.82	1.76	2.16	0.97	0.81	metal
72		6.3.5	1.72	1.7	2.16	0.99	0.79	metal
73		6.3.7	1.52	1.49	1.94	0.98	0.77	metal
74		6.3.8	0.37	0.34	0.37	0.92	0.92	metal
75		6.3.9	1.52	1.46	1.94	0.96	0.75	metal
76		6.3.10	1.04	1.01	1.94	0.97	0.52	metal
77		6.3.11	1.26	1.32	2.16	1.05	0.61	metal
78		6.3.12	1.39	1.34	2.16	0.96	0.62	metal
79		6.3.13	1.45	1.32	2.16	0.91	0.61	metal
80	8	7.3.1	0.48	0.46	0.55	0.96	0.84	metal
81		7.3.2	1.57	1.16	1.64	0.74	0.71	metal
82		7.3.3	0.97	1.34	2.84	1.38	0.47	metal
83		7.3.4	2.06	2.02	3.61	0.98	0.56	metal

Note: The "Source ref" identifies the paper in References above, from which the case was taken. The "Table entry" gives the Table No. in that paper, and the item within it, where the full case description can be found.

wall constructions, 3 wood-plank walls, 3 wood-framed roofs, 39 masonry walls, and 24 metal-framed walls.

For 45 of these 84 cases, viz. all wood and metal framed cases, new calculations of R-values were made in accordance with the set of 11 rules given here. The other 39 cases (Trethowen 1986, van Geem 1986) were all masonry walls, and the R-value calculation of those cases in the original papers are deemed to have fully followed these rules, and so were not recalculated.

Table 1 summarizes these 84 structures, with the source from which they were collected, including the location in those sources, and gives values of the thermal performance. These values include the measured R-values, the calculated R-value using the rules above, and the "ideal" R-value achievable in the absence of any thermal bridging. Table 1 does not include a full description of the structures. Because of the amount of detail required, it is not practicable to do so in this paper, which aims mainly to test the quality of calculations made by the method. To verify the results in this paper, reference to the original papers will be necessary.

## RULES

The following set of 11 rules, in three groups, are from our standard laboratory practice, and define the way in which the calculations of Table 1 have been made. They have close similarity to those implied in the 1993 *ASHRAE handbook—Fundamentals* (ASHRAE 1993). Some of these rules are well established, but there are clear variations in practice in how the ASHRAE rules are interpreted, and so all rules have been included for completeness. For the most part, references are not given in the definitions below; rules 1 through 4 are interpretations which are needed to reach a decision about what the thermal structure is; rules 5 and 7 simply ask for heat transfer data to be selected for the actual conditions; rule 6 comes from Trethowen 1988; rules 8 and 9 are standard interpretations of the formulae; rule 10 is from Trethowen 1988; and rule 11 is based on data from Trethowen 1988. An example of a complete calculation is given in Figure 5.

The basic concept is that any structure is made of one or more layers, which lie parallel to the outer faces of the structure. These layers may be homogenous as below in Rule 1, or they may have discontinuities, called "thermal bridges". The different face areas which these discontinuities project on to are called zones. In common with trends in recent years, air spaces within a structure are called cavities if they lie in the plane of the structure as layers, and edge gaps if they form thermal bridges.

## DEFINING THE LAYERS

**Homogenous Layer** "Homogenous" means that the thermal conductivity or conductance of the layer can be meaningfully expressed in a single value, and that such data are available. The term includes: fibrous or foamed insulating materials, masonry units which have been previously measured (these might be internally bridged, but that is taken to be

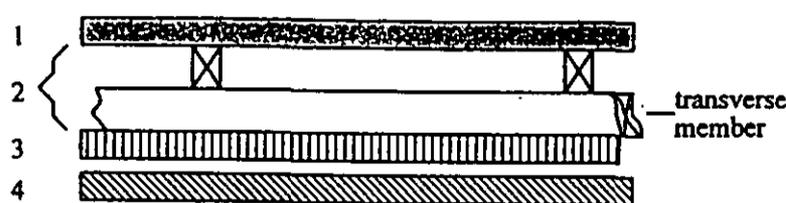


Figure 1 Illustrating layers.

reflected in the measurement), and beveled planks or boards which have been previously calculated or measured.

**Thermally Bridged Layer** A "bridged" layer is any layer which is not effectively homogenous (e.g., layer 2 in Figure 1 or other edge gaps, purlins, furring strips, structural elements, discontinuities).

**Limit of Bridged Layer** The bridged layer is defined to end at the nearest face of the next (homogenous) layer (e.g., bottom face of layer 1 and upper face of layer 3 in Figure 1), unless that next layer is insulating, when the insulating layer should be included. (Where there is any doubt as to whether the layer is "insulating," both alternatives should be considered.) If the next layer is an air cavity, the bridged layer ends outside that air cavity.

**Adjacent Zones** Two bridged layers are never immediately adjacent—there must be a separating layer, even if it is only a membrane. (If this happens in an attempted classification, then the adjacent bridged zones should be regrouped into a single zone, e.g., A frame with studs and nogging of same section, with an insulation edge gap, has three zones within the bridged layer.) Insulation only, stud or nog only (nogs are transverse frame members between studs), edge gap only A frame with studs and furring strips, insulated between studs, with a membrane between studs and furring strips, has two bridged layers, each with two zones:

- Layer 1
  - insulation only
  - stud only
- Layer 2—cavity only
  - furring only

## THERMAL RESISTANCES

**Thermal Properties** The thermal properties for each layer must be assessed for the conditions in the particular construction, e.g., rated at a suitable mean temperature and moisture content.

- If the material of a layer is compressed, both conductivity and thickness change. The resultant layer R-value needs to be measured or assessed to reflect the actual performance delivered in the condition as installed.
- If a layer of material is compressed locally over nogs, purlins, or other members, the R-value will be reduced, and there will be corner gaps. Ideally, the corner gaps form a further zone in the bridging calculation, and should be treated that way.
- reflective surfaces on the "warm" side of a cavity should be rated according to the 1993 *ASHRAE handbook—Fun-*

*damentals.* For reflective surfaces on the "cold" side, trace condensation may switch the cavity R-value between the reflective value and that for a nonreflective cavity (Bassett 1984).

**Contact Resistance** Where two layers meet, there is usually a very small "contact resistance." Except for facings in contact with metal frames, this contact resistance is always too small to be of interest in building construction. Where no better data are available, the contact resistance for normal metal framed walls is taken as  $0.03 \text{ m}^2 \cdot \text{C}/\text{W}$  ( $0.17 \text{ ft}^2 \cdot \text{h} \cdot \text{F}/\text{Btu}$ ), as suggested in Trethowen 1988. For poorer quality of physical fit, higher values will be appropriate.

**Surface Resistances** The two outer surfaces will have some combined thermal resistance which, in reality, varies continuously with weather conditions. However, the values are small and are given standard values for rating purposes. In New Zealand the standard value is  $0.12 \text{ m}^2 \cdot \text{C}/\text{W}$  ( $0.68 \text{ ft}^2 \cdot \text{h} \cdot \text{F}/\text{Btu}$ ), because the climate is rather windy: ASHRAE uses a value of  $0.15 \text{ m}^2 \cdot \text{C}/\text{W}$  ( $0.85 \text{ ft}^2 \cdot \text{h} \cdot \text{F}/\text{Btu}$ ).

## CALCULATION PROCEDURE

**Thermal Zones** Calculation of the area-weighted mean R-value of a thermally bridged layer requires the projected areas (1, 2, 3, 4 ...) of all the different zones, and the corresponding partial R-values ( $R_1, R_2, R_3 \dots$ ), to be estimated. These areas are then expressed as a fraction of the area of the basic frame module.

$$A_T = (A_1 + A_2 + A_3 + \dots) \quad (1)$$

then, for each region, the fraction of total area is given by:

$$f_1 = A_1/A_T, f_2 = A_2/A_T, \dots \quad (2)$$

whilst the resistances are  $R_1, R_2, R_3 \dots$  and the combined resistance of the thermally bridged layer  $R_b$  is

$$R_b = 1 / [f_1/R_1 + f_2/R_2 + f_3/R_3 + \dots] \quad (3)$$

**Multiple Bridged Zones** If there is more than one (separated) bridged layer, then the equivalent R-values  $R_{bridge1}, R_{bridge2} \dots$  for each is first evaluated independently as in Rule 8, and then the remaining R's of all other homogenous layers and surfaces are added.

$$R_T = \sum R_{homogenous} + R_{bridge1} + R_{bridge2} + \dots \quad (4)$$

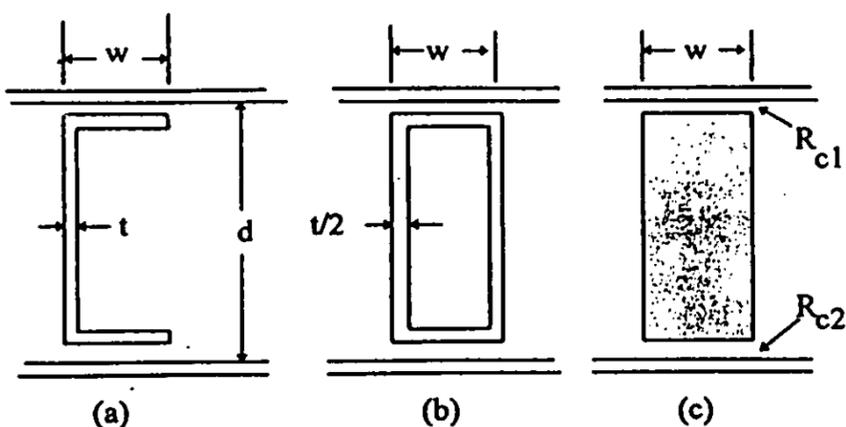


Figure 2 Transformation to a standard shape.

**Metal Frames** The whole metal frame (especially the flanges), would be expected to approach an isothermal state. It can thus be replaced by a notional "enclosing equivalent solid rectangle" of modified conductivity as in Figure 2. The shapes (a) or (b) and other variations are transformed to shape (c) by modifying the properties in the manner of Equation 5.

The series resistance of unit area of metal of conductivity  $k_m$  is  $d/k_m$ . The "equivalent resistance" of the equivalent shape is  $(w/t) \times (d/k_m)$ . The "effective resistance" of the whole metal frame is:

$$R_e = (w/t \times (d/k_m) + R_{c1} + R_{c2}) \quad (5)$$

**Convective Bridging** This item is intended to include only those situations where convective air flow can circulate around an insulating layer, viz. there are edge gaps, and cavities on both sides of the insulant. (The case of convection within the insulation is not covered here, see, e.g., Wilkes et al. 1991).

This condition is assumed to occur with all insulants (even in moderate winters over  $0^\circ\text{C}$ ), whenever there is an air cavity on both faces of the insulant, and

TABLE 2  
Edge Tolerance for Convective Bridging (ex 11)

Location	Edge Gap Tolerance
Ceilings	Any edge gaps exceed 4 mm (0.16 in.) in width
Walls	Any edge gaps exceed zero

Two (rough) alternate rules for this case (with same effect when  $d \sim 3\text{mm}$  (0.12 in.)) are:

- (a) to multiply the actual insulant layer R-value by 0.5,
- (b) to use Equation 6, which is a simple fit to the data in Trethowen 1991:

$$R_v = (0.8)^d \times R_{vo} \quad (6)$$

where

$R_v$  = effective resistance

$d$  = width of edge gap in mm (less the tolerance above)

$R_{vo}$  = original resistance of insulant

## RESULTS

Table 1 identifies the cases and shows the principal results of this comparison, over a range of  $0.3 < R < 3.0 \text{ m}^2 \cdot \text{C}/\text{W}$  ( $1.7 < R < 17 \text{ ft}^2 \cdot \text{h} \cdot \text{F}/\text{Btu}$ ). Figure 3 illustrates the results in Table 1 by directly comparing the calculated values obtained using the procedure above to the published R-values. Figure 4 shows the same data redrawn to present the "error", i.e., difference between measured and calculated values, case by case, against the measured value. Table 8 shows the distribution of errors.

In Table 1, the case numbers are arbitrary reference numbers. The "source" references are from the list of References, with the "Table" references being the internal references in those papers. The "calculated" R-values have been assessed by the Rules given above, and the "measured" values are from

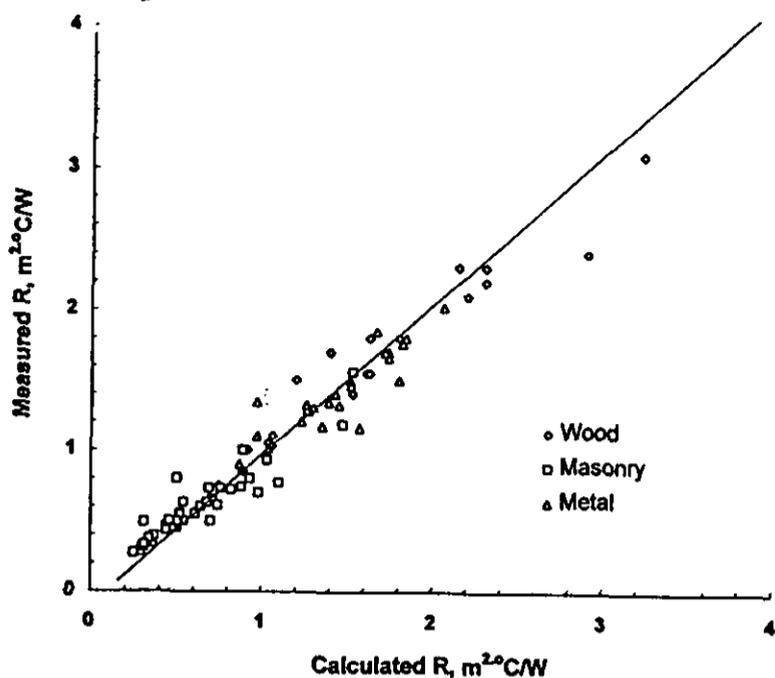


Figure 3 Measured vs. calculated R-values  $m^2 \cdot ^\circ C/W$ .

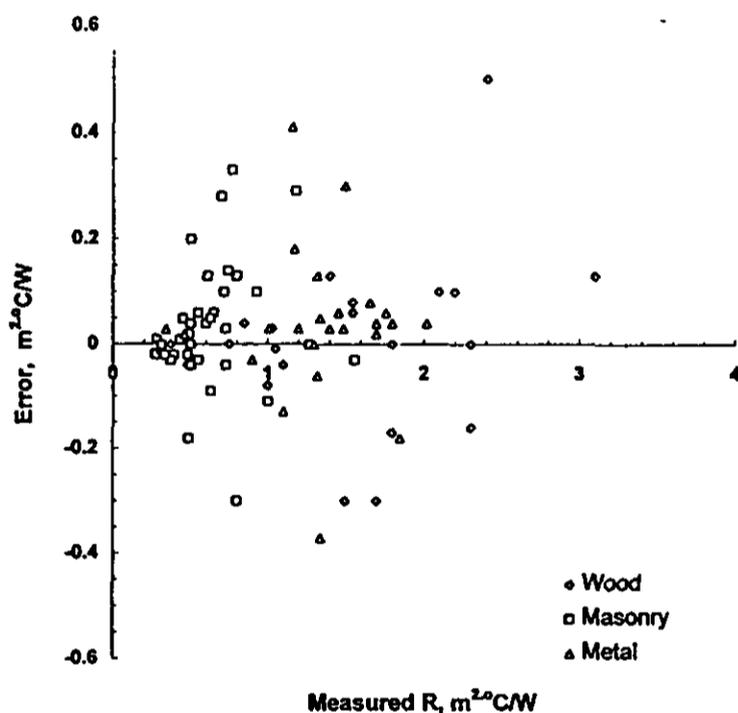


Figure 4 Difference between calculated and measured R-values.

the original papers. The "ideal" values are calculated in a totally naive way, with no allowances for thermal bridging. The surface coefficients for the calculated and measured values are the same within each case.

The fit for the different material types is of similar quality, although masonry structures in this set happen to have had lower insulation values. The difference between measured and calculated values appears not to be correlated with the value, but to have an absolute uncertainty range, which seems to be similar for all the construction groups. For the whole data set, the differences in R-values are less than  $0.1 m^2 \cdot ^\circ C/W$  ( $0.57 ft^2 \cdot h \cdot ^\circ F/Btu$ ) for 81% of cases, less than  $0.2 m^2 \cdot ^\circ C/W$  ( $0.57 ft^2 \cdot h \cdot ^\circ F/Btu$ ) for 90% of cases, and less than  $0.5 m^2 \cdot ^\circ C/W$  ( $2.8 ft^2 \cdot h \cdot ^\circ F/Btu$ ) for 100% of cases. If expressed as a percentage of the actual

R-value then, as expected, the percentage error falls as the R-value increases, and is within  $\pm 5\%$  for some 45% of these cases; within  $\pm 10\%$  for 75% of cases, and poorer than  $\pm 20\%$  in only 11% of cases. Some of these structures include workmanship defects such as edge gaps, concrete or metal thermal bridges, or other features including continuous or discontinuous thermal breaks, furring strips, special folds, or cut-outs in metal or concrete. In other words, they include a variety of real-world structures.

The discussion section reviews specific features of these calculations, and the extent to which their effect is properly reflected in the measured and calculated R-values.

## DISCUSSION

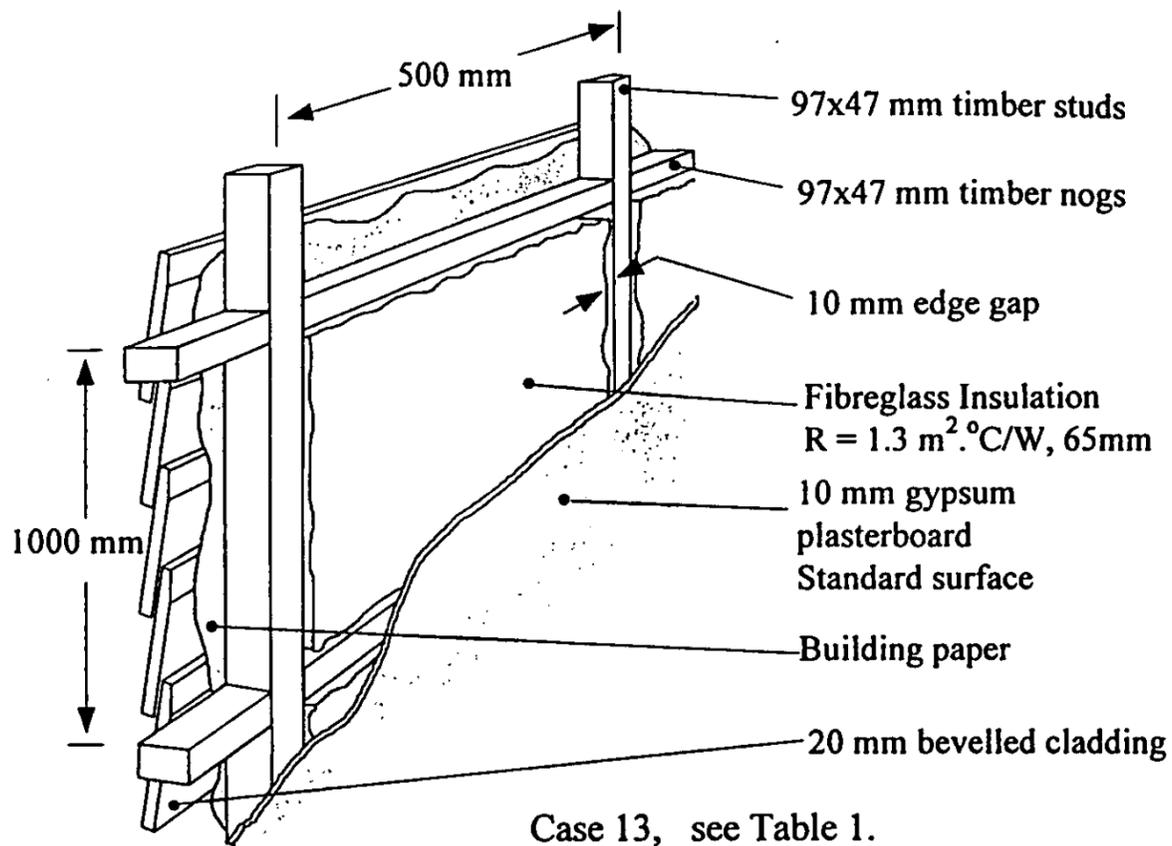
A feature of the results from previous publications and summarized here, is that the errors between calculated and measured results tends to be of difference type rather than of ratio.

There are seven results in Table 1 in which the prediction error exceeds  $0.2 m^2 \cdot ^\circ C/W$  ( $2.2 ft^2 \cdot h \cdot ^\circ F/Btu$ ), compared to the measured value. Prediction errors are to be expected in an approximate method such as this. Case 1 has the largest error ( $+0.5 m^2 \cdot ^\circ C/W$ ) ( $2.8 ft^2 \cdot h \cdot ^\circ F/Btu$ ), and this may be attributable to uncertainties arising from the small sample size in that test (only  $600 mm \times 600 mm$ ) ( $24 in. \times 24 in.$ ). Cases 6 & 7 (error  $-0.3 m^2 \cdot ^\circ C/W$ , ( $-1.7 ft^2 \cdot h \cdot ^\circ F/Btu$ )) are part of a series in which an increasing gap width was inserted into otherwise constant insulation. In these two cases, the gap widths reached  $\sim 50 mm$  (2 in.) and  $80 mm$  ( $3 \frac{1}{4} in.$ ) respectively, and the method seems to underestimate R-values in this condition.

In masonry cases 53 ( $+0.28 m^2 \cdot ^\circ C/W$ ) ( $1.6 ft^2 \cdot h \cdot ^\circ F/Btu$ ) and 55 ( $0.38 m^2 \cdot ^\circ C/W$ ) ( $+2.2 ft^2 \cdot h \cdot ^\circ F/Btu$ ) the causes are not fully clear, but case 53 uses "EPS inserts" in the blocks, and this has been found to be a less reliable method of insulating as there is significant risk that imperfect fitting will permit convective flow around the insert, lowering the measured R-value. In case 55 which uses perlite cavity fill, it may be conjectured that the perlite fill was incomplete, also lowering the measured R-value. This did not seem to occur in the sister cases, perhaps by chance.

Cases 81 ( $+0.4 m^2 \cdot ^\circ C/W$ ) ( $2.3 ft^2 \cdot h \cdot ^\circ F/Btu$ ) and 82 ( $-0.37 m^2 \cdot ^\circ C/W$ ) ( $2.1 ft^2 \cdot h \cdot ^\circ F/Btu$ ) are not easily explained on the basis of sample description. In particular, it is difficult to explain how case 82 should have a significantly higher measured R-value than case 81. The Isothermal Planes method, and also general laboratory experience suggests that, the reverse might be expected. Strzepek has since defended his result using an argument for case 81 (and 83), that the metal fixings at  $300 mm$  (12 in.) through the insulating sheathing undermined the wall performance. This is conceivable but not yet verified.

A substantial message from these "outlier" cases is that in practical construction, even in the laboratory, some apparently minor assembly variations can significantly alter the performance. In the author's laboratory it has been noted that the "same" wall, re-erected from the same members to the same



From Rules 1-4, the region between the frame side of the gypsum plaster board and the building paper, is the region considered to be thermally bridged.

**R-Value Calculation, by Isothermal Planes method:**

Area ratios: framing:  $0.47 \cdot (500+1000) / (500 \cdot 1000) = 0.14$   
 insulation edge gap:  $0.01 \cdot (500+1000) / (500 \cdot 1000) = 0.03$   
 insulation :  $1 - 0.141 - 0.03 = 0.83$   
 Thermal resistances framing :  $0.097 / 0.13 = 0.75$   
 insulation edge gap :  $= 0.17$   
 insulation :  $1.3 + 0.17 = 1.47$   
 gypsum board :  $0.01 / 0.24 = 0.04$   
 Redwood cladding :  $0.020 / 0.13 = 0.15$   
 partial cavity between  
 cladding and sheathing:  $= 0.14$   
 internal + external surfaces:  $= 0.12$

**Thermal bridged region**

$$R_b = 1 / [ 0.14 / 0.75 + 0.03 / 0.19 + 0.83 / 1.47 ] = 1.08$$

**Total thermal resistance**

Bridged region = 1.08  
 Gypsum lining = 0.04  
 Cladding = 0.15  
 0.14  
 Surfaces = 0.12  
 Total = 1.53 m<sup>2</sup>·°C/W

(NB. By Parallel Flow method,

$$R_{\text{total}} = 1 / [ 0.14 / (0.45 + 0.75) + 0.03 / (0.45 + 0.17 + 0.83 / (0.45 + 1.47)) ] = 1.68 \text{ m}^2 \cdot \text{°C/W} )$$

( Ignoring thermal bridging,

$$R_{\text{total}} = 1.92 \text{ m}^2 \cdot \text{°C/W} )$$

**Measured Value = 1.4 m<sup>2</sup>·°C/W**

*Figure 5 Example calculation by the isothermal planes method.*

ification, can give such variations. These influences will just as important in high precision calculations by finite element modelling. The assignment of an R-value has to be regarded as typical or average, not as an assured rating.

An important item is the behavior of metal framed structures, and the way that these have been treated here. Metals are highly conducting that even thin cross sections can carry large amounts of heat. A 50-mm (2-in.) depth of steel will have thermal resistance of only  $0.001 \text{ m}^2 \cdot ^\circ\text{C}/\text{W}$  ( $0.0057 \text{ ft}^2 \cdot \text{h} \cdot ^\circ\text{F}/\text{Btu}$ ), and would carry  $100 \text{ W}/\text{m}^2$  ( $32 \text{ Btu}/\text{h} \cdot \text{ft}^2$ ) with only  $0.2 \text{ }^\circ\text{C}$  ( $0.2 \text{ }^\circ\text{F}$ ) temperature difference. Aluminum would carry  $95 \text{ W}/\text{m}^2$  ( $95 \text{ Btu}/\text{h} \cdot \text{ft}^2$ ) with this temperature difference.

For this reason, any piece of metal in a structure is nearly isothermal, i.e., it will have similar temperature at all points. Where it contacts other members, especially the facing layers, there may be substantial temperature gradients across the junction. Heat conduction from face to face through the metal is affected by the length of the metal web, but the ability of this heat to pass between the metal and the facing would appear to be controlled by the width of the flange. The metal shapes are often quite complex, but are expected to have only slight temperature differences, because of their high conductivity. These facts are consistent with the indication of Table 4 that the details of the metal cross section are unimportant, and that the transformation to a rectangular shape as in Rule 9 (Figure 2) will give a sufficient description of the heat transmission. For example, the series thermal resistance of 80 mm (3.15 in.) deep steel web of conductivity  $50 \text{ W}/\text{m} \cdot ^\circ\text{C}$  ( $345 \text{ Btu}/\text{in} \cdot \text{h} \cdot \text{ft}^2 \cdot ^\circ\text{F}$ ) would be  $0.0016 \text{ m}^2 \cdot ^\circ\text{C}/\text{W}$  ( $0.009 \text{ ft}^2 \cdot \text{h} \cdot ^\circ\text{F}/\text{Btu}$ ), but if the steel were to be transformed to the shape (c) of Figure 2, the equivalent resistance would become  $0.03$  to  $0.04 \text{ m}^2 \cdot ^\circ\text{C}/\text{W}$  ( $0.17$  to  $0.23 \text{ ft}^2 \cdot \text{h} \cdot ^\circ\text{F}/\text{Btu}$ ), depending on the exact dimensions.

A second extra provision is needed to deal with thermal contact quality. When building materials are assembled together they never fit perfectly. The contact resistance between them is usually small, and in most normal cases, is of no consequence, as typical contact resistances in building may be, say, only  $0.03 \text{ m}^2 \cdot ^\circ\text{C}/\text{W}$  ( $0.17 \text{ ft}^2 \cdot \text{h} \cdot ^\circ\text{F}/\text{Btu}$ ) (Trethowen 1988). However, if there were a contact resistance even of only  $0.03 \text{ m}^2 \cdot ^\circ\text{C}/\text{W}$  ( $0.17 \text{ ft}^2 \cdot \text{h} \cdot ^\circ\text{F}/\text{Btu}$ ) at each face, the equivalent steel web resistance of  $R = 0.04 \text{ m}^2 \cdot ^\circ\text{C}/\text{W}$  ( $0.23 \text{ ft}^2 \cdot \text{h} \cdot ^\circ\text{F}/\text{Btu}$ ) would increase to  $R = 0.1 \text{ m}^2 \cdot ^\circ\text{C}/\text{W}$  ( $0.57 \text{ ft}^2 \cdot \text{h} \cdot ^\circ\text{F}/\text{Btu}$ ). The result of these several effects is that the apparent R-value of the steel web element is about 60 times the original estimate of 0.0016.

In masonry construction there can be significant differences between the R-value of the masonry units and that of a wall made from those units. Key factors with high-R blocks are the mortaring system used, and the filling of hollow cores with either cement grout for reinforcing steel or with insulant for thermal efficiency. Core filling tends to have only small effect on the R-value of high density masonry (over say,  $2,000 \text{ kg}/\text{m}^3$ ) ( $125 \text{ lb}/\text{ft}^3$ ), because the conductance of the original masonry material is high enough that the core treatment is

overshadowed. Even core grouting does not then have a strong effect (Trethowen 1986).

However, for lower densities, core filling has an increasing effect. In highly insulated masonry units assembled using traditional mortaring practices, thermal bridging through the mortar system can limit the overall R-value of a wall to about  $0.7 \text{ m}^2 \cdot ^\circ\text{C}/\text{W}$  ( $4 \text{ ft}^2 \cdot \text{h} \cdot ^\circ\text{F}/\text{Btu}$ ) (Trethowen 1986), because of the influence of even minor mortar spill, and the presence of convective openings. To get the proper potential from such units, novel mortaring systems may be required.

The calculation of masonry R-values can be done using the Isothermal Planes method, but often measurement is required to ensure reliability of rating. There are several difficulties in any calculation, but particularly for masonry: the geometry is often complex in three dimensions: mortar spill often extends beyond the expected parts of the block faces: there is often a considerable degree of interconnection between the cavities.

### Quality of Insulation Fit

Case 13 had  $1,000 \times 500 \times 97 \text{ mm}$  ( $39.4 \text{ in.} \times 19.7 \text{ in.} \times 3.9 \text{ in.}$ ) timber frame with bevelled cedarwood planking and 80 mm (3.15 in.) fiberglass batt insulation, fitted with approximately 10 mm (0.39 in.) wide-edge gaps around all edges of the fiberglass. The R-value of the insulant was estimated at 1.3 (7.4) plus air gap of 0.17 (0.9), totaling  $\sim 1.47 \text{ m}^2 \cdot ^\circ\text{C}/\text{W}$  ( $8.4 \text{ ft}^2 \cdot \text{h} \cdot ^\circ\text{F}/\text{Btu}$ ).

TABLE 3  
Effect of Gaps

Edge Gap	Calculated	Measured
	R-value $\text{m}^2 \cdot ^\circ\text{C}/\text{W}$ ( $\text{ft}^2 \cdot \text{h} \cdot ^\circ\text{F}/\text{Btu}$ )	
With 10 mm edge gap (13)	1.53 (9.1)	1.4 (7.9)
With no edge gap	1.8 (10.2)	—

There was no measured value reported in this particular reference with no edge gap. But for this type of structure, the calculated value for the no-edge-gap case would be expected from experience to be fairly reliable.

### Shape of Steel Stud (see Rule 9)

Cases 62, 63, and 66 (Trethowen 1988) compare three steel stud sections. In case 62, C-section studs were 0.5 mm (0.02 in.) thick at 400 mm (15.7 in.) centers, and the installed insulant had a resistance of  $1.94 \text{ m}^2 \cdot ^\circ\text{C}/\text{W}$  ( $11 \text{ ft}^2 \cdot \text{h} \cdot ^\circ\text{F}/\text{Btu}$ ). In case 63, Z-section studs were 0.91 mm (0.036 in.) thick at 410 mm (16.1 in.) centers with cutouts in the web, and installed insulant had resistance estimated at  $1.3 \text{ m}^2 \cdot ^\circ\text{C}/\text{W}$  ( $7.4 \text{ ft}^2 \cdot \text{h} \cdot ^\circ\text{F}/\text{Btu}$ ). In case 66, S-section studs were double 1.6 mm (0.063 in.) roll-formed steel with 50 mm (2 in.) foamed in-situ PU insulation of  $\sim 37 \text{ kg}/\text{m}^3$  ( $590 \text{ lb}/\text{ft}^3$ ), estimated as having an R-value of  $\sim 2.4 \text{ m}^2 \cdot ^\circ\text{C}/\text{W}$  ( $13.6 \text{ ft}^2 \cdot \text{h} \cdot ^\circ\text{F}/\text{Btu}$ ). Table 4 indicates that the shape of the steel section does not have a marked effect.

**TABLE 4**  
**Effect of Shape of Steel Studs**

Section Shape	Calculated R-value m <sup>2</sup> °C/W (ft <sup>2</sup> .h.°F/BTU)	Measured
C-Section (62)	1.35 (7.7)	1.17 (6.6)
Z-Section (63)	1.23 (7.0)	1.20 (6.8)
S-Section (66)	1.30 (7.4)	1.30 (7.4)

**Effect of Contact Resistance  
with Steel Stud (see Rule 8)**

From cases 73 and 77 (Carson et al. 1993), we compare a panel before and after reassembly from the same components, with two differences—first, the reassembly was done so that the mean estimated gap between steel frame and facings was reduced from 1.5 mm to 2.0 mm (0.059 in. to 0.079 in.) to less than 1.0 mm (0.04 in.) (i.e., the contact resistance drops from an estimated 0.06 to 0.03 m<sup>2</sup>.°C/W (0.34 to 0.17 ft<sup>2</sup>.h.°F/Btu)). Second, the stud spacing was halved from 1,000 mm to 500 mm (39.4 in. to 19.7 in.).

The combined effect is in Table 5.

The effect of closer stud spacing and that of contact gap are estimated to have approximately equal effect. Agreement with the measured values is good in both cases.

**Effect of Steel Thickness (see Rule 8)**

From cases 77 and 78 (Carson et al. 1993), identical except for steel thickness, we get Table 6. For a change in steel thickness from 1.2 mm to 0.8 mm, the effective steel resistance goes from 0.041 to 0.059 (0.23 to 0.34) and overall R-value difference is calculated to be only 0.06 to 0.07 m<sup>2</sup>.°C/W (0.34 to 0.40 ft<sup>2</sup>.h.°F/Btu). Table 6 indicates that the steel thickness has only limited influence.

**Effect of Omitting Insulant within Steel (see Rule 8)**

From cases 73 and 75 (Carson et al. 1993), identical except that insulant filled the space within the C-section steel, or was absent from that zone, we get Table 7.

Table 7 indicates that it is not important to insulate inside the metal section.

**TABLE 5**  
**Effect of Contact and Stud Spacing**

Stud Spacing and Contact Gap	Calculated R-value m <sup>2</sup> °C/W	Measured (ft <sup>2</sup> .h.°F/BTU)
R-value, 1000 mm, 2 mm contact gap (73)	1.52 (8.6)	1.49 (8.5)
R-value, 500 mm, 1 mm contact gap (77)	1.26 (7.2)	1.32 (7.5)

**TABLE 6**  
**Effect of Steel Thickness**

Steel Thickness	Calculated R-value m <sup>2</sup> °C/W	Measured (ft <sup>2</sup> .h.°F/BTU)
1.2 mm steel (0.047") (77)	1.26 (7.2)	1.32 (7.5)
0.8 mm steel (0.031") (78)	1.39 (7.9)	1.34 (7.6)

**TABLE 7**  
**Effect of Omitting Insulant within Stud**

Cavity within Steel Stud	Calculated R-value m <sup>2</sup> °C/W	Measured (ft <sup>2</sup> .h.°F/BTU)
Space inside steel insulated (73)	1.52 (8.6)	1.49 (8.5)
Space inside steel empty (75)	1.52 (8.6)	1.46 (8.3)

**TABLE 8**  
**Difference between Measured and Calculated R-Values**

No.	Difference (measured-calculated) R-value m <sup>2</sup> °C/W (ft <sup>2</sup> .h.°F/BTU)										
	-0.5	-0.4	-0.3	-0.2	-0.1	0.0	0.1	0.2	0.3	0.4	0.5
	1	1	2	2	9	42	17	6	0	4	0

Overall distribution of prediction errors. Table 8 shows the distribution of errors in the predicted results, taking the measured values as true. This indicates that there are some outlier values, but that the predominant agreement is within 0.1 to 0.2 m<sup>2</sup>·°C/W.

## CONCLUSIONS

This paper considers the validity of the isothermal planes method by comparing measured vs. calculated values. A data set of 84 published cases of measured R-values from major publications has been used for this purpose. This comparison indicates that a satisfactory fit for engineering purposes can be obtained. Specific justification for certain of these rules has been presented by case comparison.

Over the whole data set the differences between measured and calculated values tended to be not correlated with the R-value, and were within ±0.1 m<sup>2</sup>·°C/W (0.57 ft<sup>2</sup>·h·°F/Btu) for 81% of the cases considered. (Some 45% of forecasts were within 5% of the corresponding measured results; 75% were within 10%, and only 11% were outside 20%). The greatest percentage differences occurred at lower R-values.

For all three main types of construction (wood framed, masonry, and metal framed), the agreement between calculated and measured values was similar. The overall mean of calculated results was within 2% of measured for all three types, and the standard deviation for individual cases was found to be 10% for wood-framed, 17% for masonry constructions, and 12% for metal-framed.

The principal step by which agreement with metal framed structures was achieved was by including allowance for tiny but realistic contact resistances between the metal frame and the facings. This effect has been separately shown to have physical reality, as well as achieving better forecasting fit.

The method needs better definition of some of its rules. The rules for the isothermal planes method used in one building research laboratory have been detailed, and were used for the calculations presented here. These rules provide specifically for a number of workmanship defects to be modeled.

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## DISCUSSION

Ned Nisson, Editor, *Energy Design Magazine*, New York, NY: What is the effect of contact resistance between steel studs and sheathing?

Harold A. Trethowen: The contact resistance between a steel frame and sheathing, and the resistance of the sheathing itself, are added. They may often have a very significant effect by adding a degree of thermal resistance between the highly con-

ducting steel and the surface facing material. In the suggested set of calculation rules offered in this paper, the thermal resistance of the sheathing is included with any contact resistance in both the "bridged" and "unbridged" heat flow paths. This gives results somewhat like the actual measured values that have been reported for steel-framed walls with sheathings.



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