





REPRINT

No. 130 (1994)

**Using Hygroscopic Damping of
Relative Humidity and Vapour
Pressure Fluctuations to Measure
Room Ventilation Rates**

M J Cunningham

From: Building and Environment,
Vol 29, No. 4, pp. 501-510, 1994



Using Hygroscopic Damping of Relative Humidity and Vapour Pressure Fluctuations to Measure Room Ventilation Rates

M. J. CUNNINGHAM*

A field study (carried out for other purposes) indicated that relatively air-tight rooms have small relative humidity fluctuations in response to fluctuating indoor temperature and outdoor vapour pressures. This effect is due to hygroscopic damping and has been quantified here to allow the derivation of a simple formula giving room ventilation rates as a function of the amplitude of sinusoidal fluctuations in external vapour pressure, and fluctuations in room vapour pressure and humidity. It is found that the room air-change rate is proportional to the room relative humidity fluctuations, and inversely proportional to the difference between the indoor and outdoor vapour pressure fluctuations, allowing for phase differences. Calculated results agree well with field data.

NOMENCLATURE

A area of room hygroscopic materials exposed to room air (m^2)
 c_e moisture concentration in the external air (kg m^{-3})
 c_o moisture concentration in the room air (kg m^{-3})
 d effective depth (m)
 D_m diffusion coefficient under mass concentration driving force ($\text{m}^2 \text{s}^{-1}$)
 F room air-change rate (s^{-1})
 h surface heat transfer coefficient ($\text{W m}^{-2} \text{°C}^{-1}$)
 k heat conductivity ($\text{W m}^{-1} \text{°C}^{-1}$)
 k_h differential sorption coefficient connecting vapour pressure and moisture content of hygroscopic room contents (see equation (7)) ($\text{Pa m}^3 \text{kg}^{-1}$)
 l_h differential sorption coefficient connecting relative humidity and moisture content of hygroscopic room contents ($\text{m}^3 \text{kg}^{-1}$)
 m_h mean hygroscopic material moisture concentration throughout the effective depth (kg m^{-3})
 m_o equilibrium moisture content corresponding to the room relative humidity (kg m^{-3})
 p_e external vapour pressure (Pa)
 p_o room vapour pressure (Pa)
 p_s room saturated vapour pressure (Pa)
 r effective resistance with vapour pressure driving forces (N s kg^{-1})
 R effective resistance with mass concentration driving forces (s m^{-1})
 R universal gas constant ($8310 \text{ J K}^{-1} \text{ kmole}^{-1}$)
 R_s surface resistance (s m^{-1})
 S moisture source strength (kg s^{-1})
 t time (s)
 t_1, t_2 time constants (s)
 T Kelvin temperature (K)
 V_h effective volume of hygroscopic material interacting with cyclic humidity driving forces (m^3)
 V_o volume of room (m^3)
 W molecular weight of water (18 kg kmole^{-1})
 θ_h phase of hygroscopic material relative humidity relative to external vapour pressure (radians)

θ_o phase of room vapour pressure relative to external vapour pressure (radians)
 θ_r phase of room temperature relative to external vapour pressure (radians)
 θ_T phase of hygroscopic material temperature relative to external vapour pressure (radians)
 θ_ϕ phase of room relative humidity relative to external vapour pressure (radians)
 ρ density (kg m^{-3})
 ϕ relative humidity
 ω angular frequency (radians s^{-1})

Subscripts

e external air
 h hygroscopic material
 o room air
 r room air
 s saturated

INTRODUCTION

A HOST of problems arises if indoor humidity levels are too high for too long. Cooler surfaces can collect condensation with the corresponding risk of surface finish and mould damage; high relative humidity is associated with health problems [1], often caused by allergens released by biota such as dust-mites and mould. Of interest here are not only the mean relative humidity levels, but also their peak values. The humidity fluctuations can be short-term, due to transient sources such as cooking, showering, etc. but there also tend to be daily and yearly cyclic fluctuations, particularly in spaces that are not controlled, influenced by periodic fluctuations in the external conditions. Hygroscopic materials within a room tend to damp relative humidity swings, so that the magnitude of these fluctuations is determined in a complex way by external climate driving forces, internal sources and heating, and hygroscopy of the room contents. Additionally, the cycling of hygroscopic moist-

*Building Physicist, BRANZ, Moonshine Road, Judgeford, Porirua, New Zealand.

ure adds latent load to any air conditioning plant within the space [2, 3].

With these concerns in mind, several authors have modelled room psychrometric conditions [2-5] and also studied absorption and desorption of moisture in the hygroscopic contents of the room [6-8]. These models often predict room conditions accurately, but sometimes underlying simplicities are concealed by the complex and numerical nature of the calculations. A room consists simply of an interior volume ventilated with external air, the cyclic air moisture content interacting with a thin layer of the room's hygroscopic materials. This suggests that fundamentally the system should be governed by one or two time constants determined by the amount of ventilation, the amount of hygroscopic material and their relative importance. This underlying simplicity is exploited in this work to allow a simple formula to be derived giving the room ventilation rate as a function of the sinusoidal fluctuations in room and external conditions and hygroscopic damping.

OBSERVATIONAL BACKGROUND

This work was motivated by the need to explain some interesting results obtained in a recent year-long field survey involving moisture measurements in a number of houses around New Zealand [9]. It was noted that those rooms known to have low ventilation rates (called "air-tight" or simply "tight" in this work) had low internal relative humidity fluctuations, while those houses known to have high ventilation rates ("air-loose" or "loose") had higher relative humidity fluctuations. Loose rooms behaved in the way expected, i.e., the water vapour pressure tended to follow the exterior vapour pressure and internal relative humidities tended to be lower as the internal temperature rose, and vice versa. However, tight rooms exhibited a totally different behaviour; not only were relative humidity fluctuations lower but they were also relatively independent of temperature, and consequently room vapour pressure tended to rise with room temperature.

These trends are illustrated in Figs 1-3 using data taken from the field survey. Ignoring short-term effects due to transient moisture sources such as showering, cooking, etc., Fig. 1 shows the performance of a loose room demonstrating that indoor vapour pressure follows outdoor vapour pressure and indoor relative humidity falls as the indoor temperature rises. Figure 2 shows the performance of a tight room demonstrating that internal relative humidity fluctuations are small and relatively independent of internal temperature while the vapour pressure rises as internal temperature rises. Figure 3 quantifies these trends by showing the correlation in each case between the internal temperature, relative humidity and vapour pressure and the external vapour pressure. (The tight room is a poorly ventilated bathroom, which explains the high relative humidity levels in this room.)

The explanation for these results lies in the hygric damping effect caused by the presence of hygroscopic materials within the room. These materials attempt to hold the relative humidity of the room constant independently of the temperature. The relative influence of these hygroscopic materials increases as the room

becomes tighter; therefore it should be possible to calculate the air-change rate within the room given the magnitude of the relative humidity fluctuations compared to that expected if there were no hygroscopic materials present.

This paper quantifies this effect by solving for the dynamic mass balance within the room under cyclic conditions, and in the presence of hygroscopic materials. This allows a formula to be derived for the air-change rate in a room given the level of the relative humidity fluctuations. It is shown that this formula gives good agreement with the air-change rates measured during the field survey.

Continuous measurement of air exchange rates in rooms is difficult and expensive, requiring the use of specialized tracer gas equipment [10]. This technique suggests one way that at least approximate values of air-change can be obtained using simple and inexpensive-to-run equipment.

It must be emphasized that this analysis is based around sinusoidally fluctuating quantities and is not the usual steady-state approach which connects moisture source strength, indoor and outdoor vapour pressures and air-change rates, viz.:

$$p_o = p_e + \frac{S}{V_o F} \frac{RT}{W}$$

This latter formula does not consider the presence of hygroscopic materials, as indeed it cannot because once at steady-state hygroscopy has no influence. This formula does however drop out as a by-product of the analysis.

THEORY

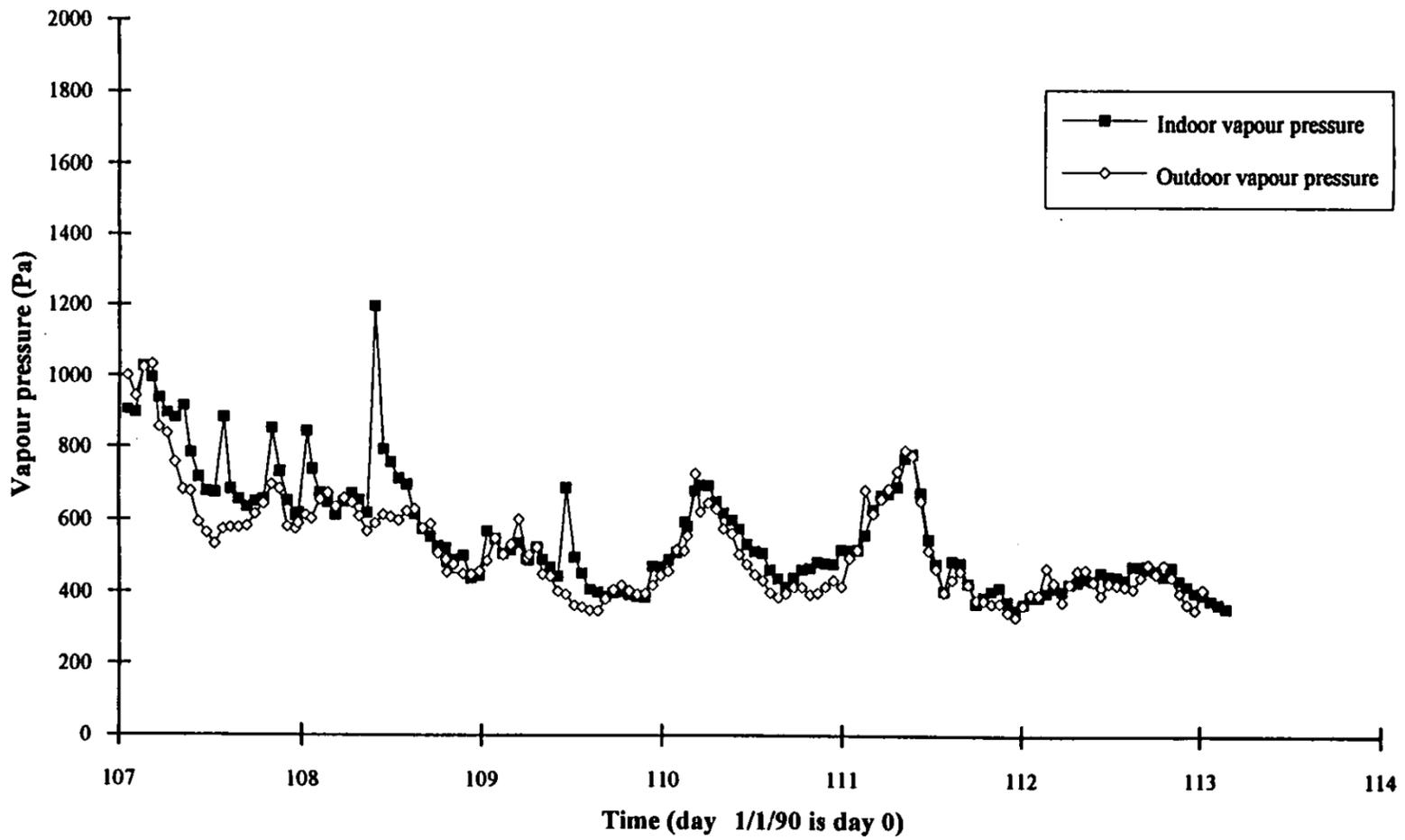
A room is modelled with a steady air-change rate of F air changes per second from and to the outdoors and in the presence of hygroscopic materials. Only constant moisture sources are considered initially. The driving forces are taken to be external vapour pressure and internal temperature, both taken as sinusoidal. The periodicity would normally be taken as one day although this is not necessary to the analysis.

These assumptions may appear quite restrictive, but in practice they are found to mirror reasonably well conditions found in the field [9], at least for periods of days at a time when the climate is mild enough that no extra heating is used. In practice there are of course many transient moisture sources, but most of these last only for a period of time that is short compared to a day (e.g. cooking, washing, showering).

Using these assumptions, the differential equations are solved which describe the mass balance of the room in terms of vapour pressure and hygroscopic material relative humidity. This in turn leads to an expression for the room ventilation rate as a function of the amplitude of the room relative humidity and the external vapour pressure.

It is well known that under cyclic conditions moisture in the air is exchanged in and out of a thin surface layer of any hygroscopic materials present in the room [4, 8]. Cunningham [8] has shown, firstly, that this layer is of thickness d given by

1(a) Vapour pressures for a loose house



1(b) Relative humidity and temperature in a loose room

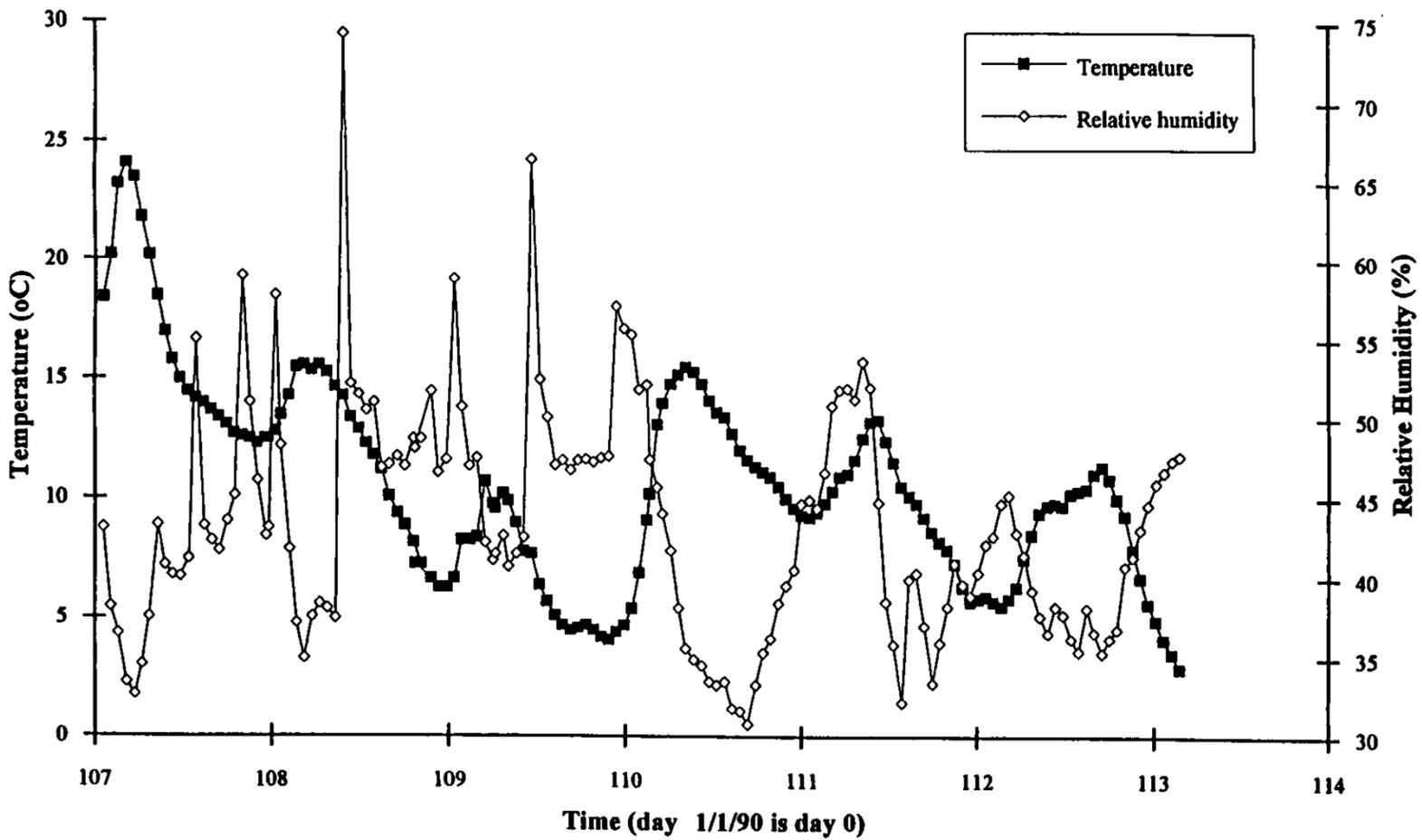
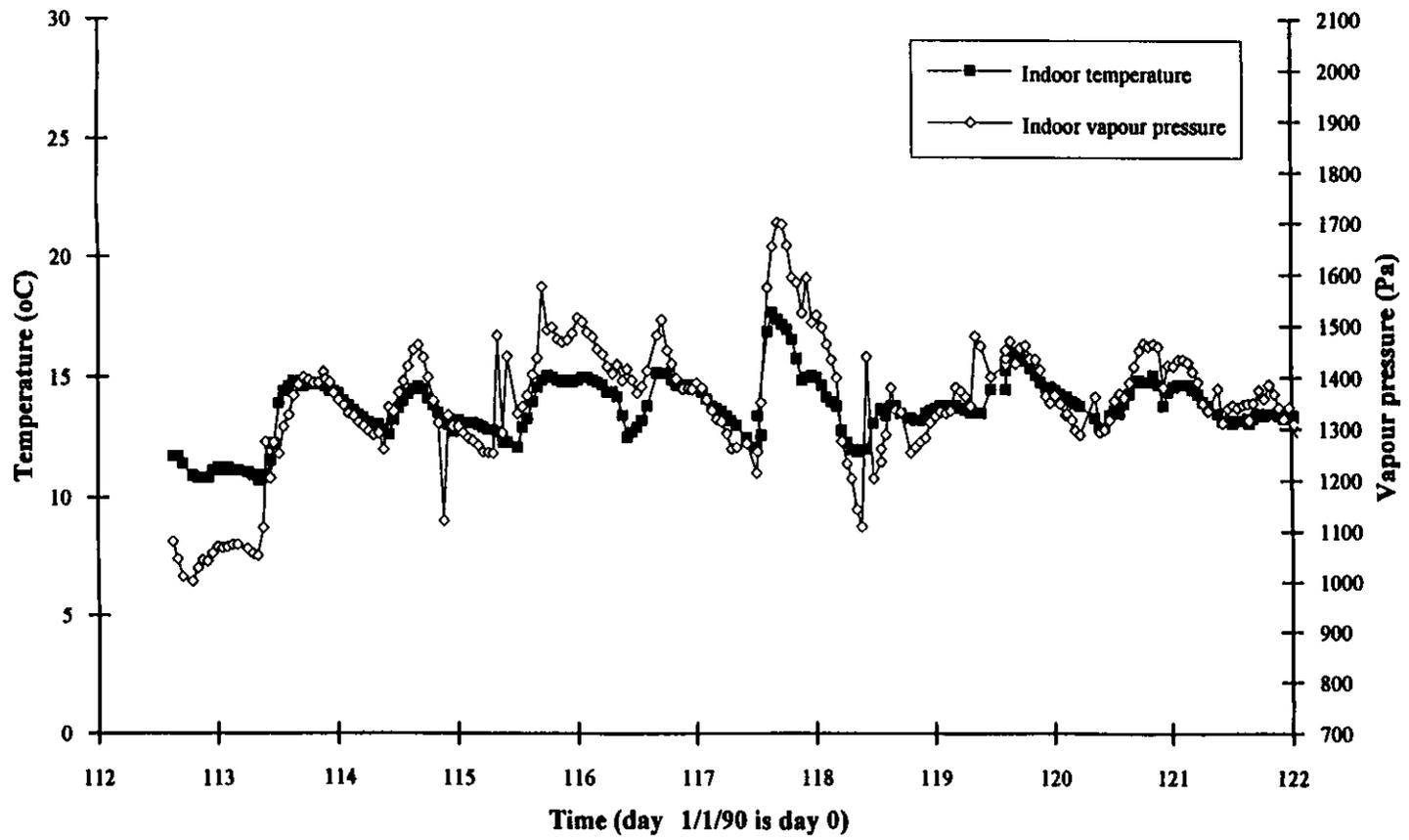


Fig. 1. Comparative fluctuations of relative humidities, vapour pressures, and temperatures in an air-loose room. Room vapour pressure follows external vapour pressure, and room relative humidity is inversely correlated with room temperature.

2(a) Temperature and vapour pressure in a tight room



2(b) Relative humidity and temperature in a tight room

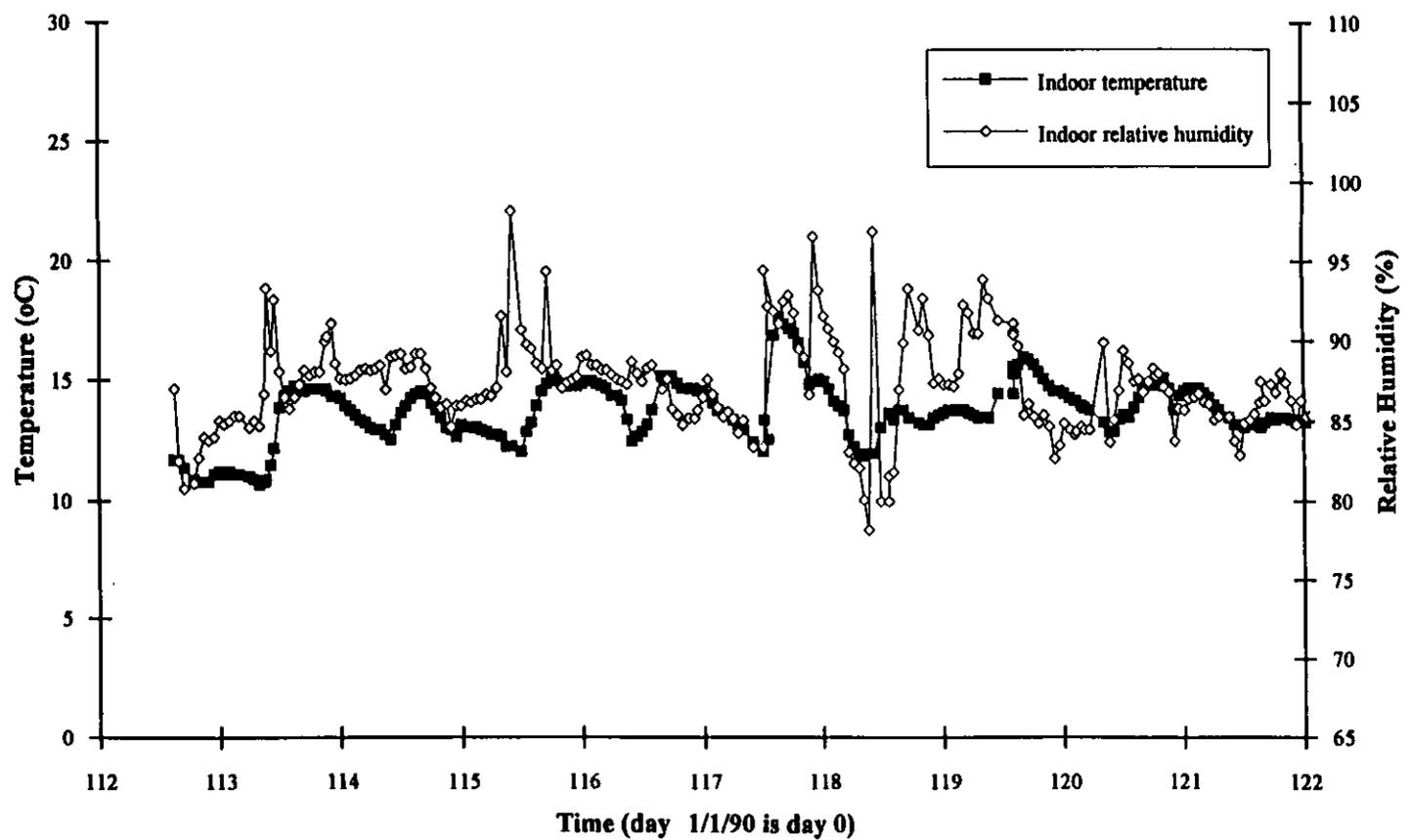


Fig. 2. Comparative fluctuations of relative humidities, vapour pressures, and temperatures in a tight room. Room vapour pressure follows room temperature and room humidity fluctuations are small and independent of room temperature.

$$d = \sqrt{\frac{D_m}{2\omega}} \quad (1)$$

where D_m is the diffusion coefficient under mass concentration driving force, and secondly, that the moisture exchange takes place through an effective resistance R of

$$R = \frac{d}{D_m} + R_s \quad (2)$$

where R_s is the surface resistance.

The effective volume V_h of hygroscopic material involved in periodic moisture exchange will be

$$V_h = 2Ad \quad (3)$$

where A is the area of hygroscopic material exposed to the room air. The factor of 2 in equation (3) follows from the definitions used to derive the effective depth d ; this is the centre point of the volume over which the moisture exchanging with the room appears to be spread, see [8].

These equations were derived in [8] using a mass exchange equation between the room air and the hygroscopic material of the form

$$V_h \frac{\partial m_h}{\partial t} = A \left(\frac{m_o - m_h}{R} \right) \quad (4)$$

where m_h is the mean moisture content through the effective depth and m_o is the equilibrium moisture content corresponding to the room relative humidity. In this work mass transfer to and from the hygroscopic materials is modelled with vapour pressure driving forces, so the appropriate equation becomes

$$V_h \frac{\partial m_h}{\partial t} = A \left(\frac{p_o - p_h}{r} \right) \quad (5)$$

The connection between equations (4) and (5) is through the sorption curve of the material. In particular it is necessary to find the connection between the effective resistances r and R for vapour pressure and moisture content driving forces respectively.

Specifically if the slope of the sorption curve l_h (or the differential sorption coefficient) is defined as

$$\frac{\Delta \phi_h}{\Delta m} = l_h$$

then since by definition

$$\Delta \phi_h = \frac{\Delta p}{p_s} \quad (6)$$

3(a) Scatter plots for a loose room

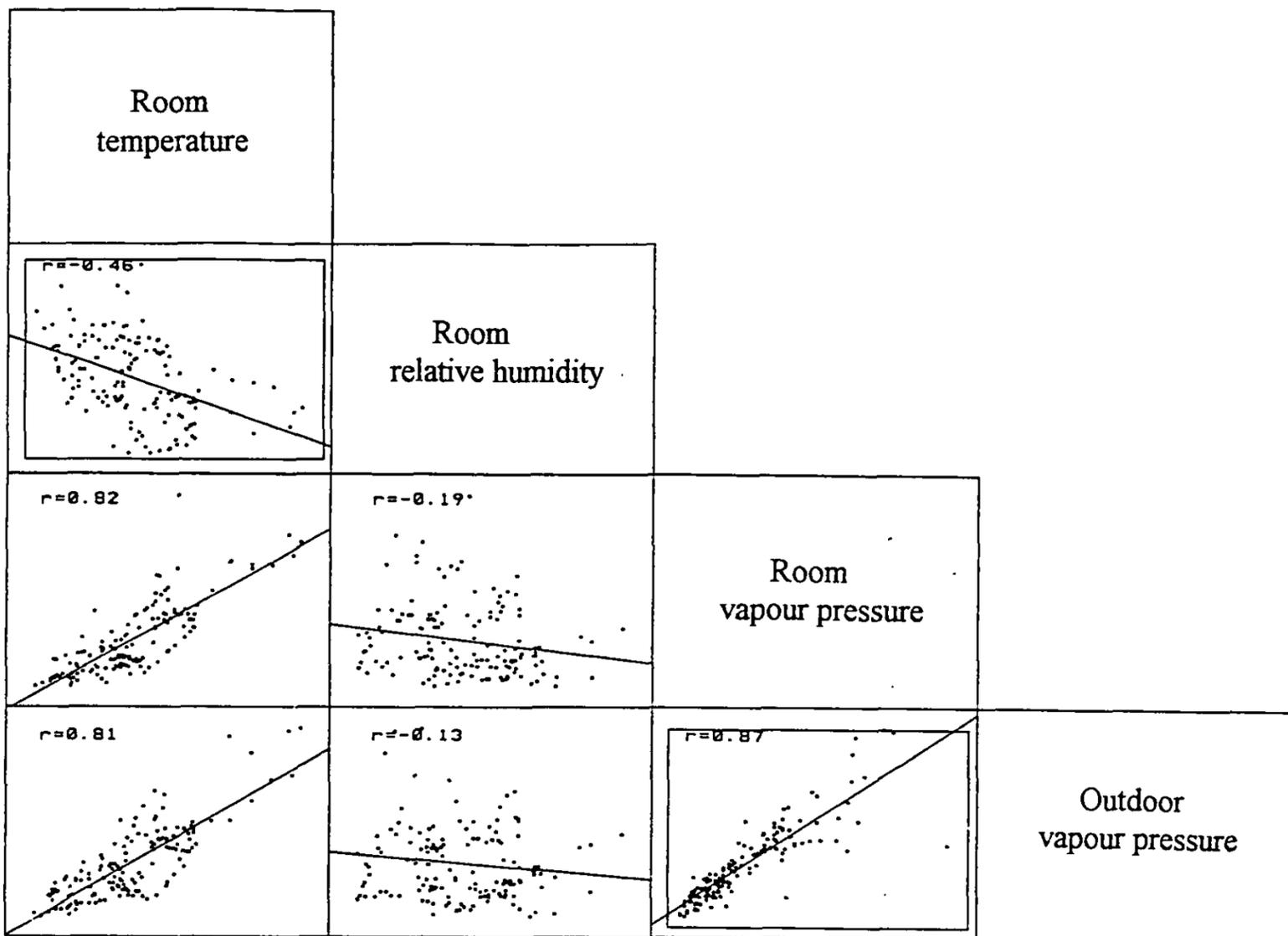


Fig. 3. Correlations between relative humidities, vapour pressures, and temperatures in air loose and tight rooms quantifying remarks made in Figs 1 and 2. Important plots are boxed. (continued overleaf)

3(b) Scatter plots for a tight room

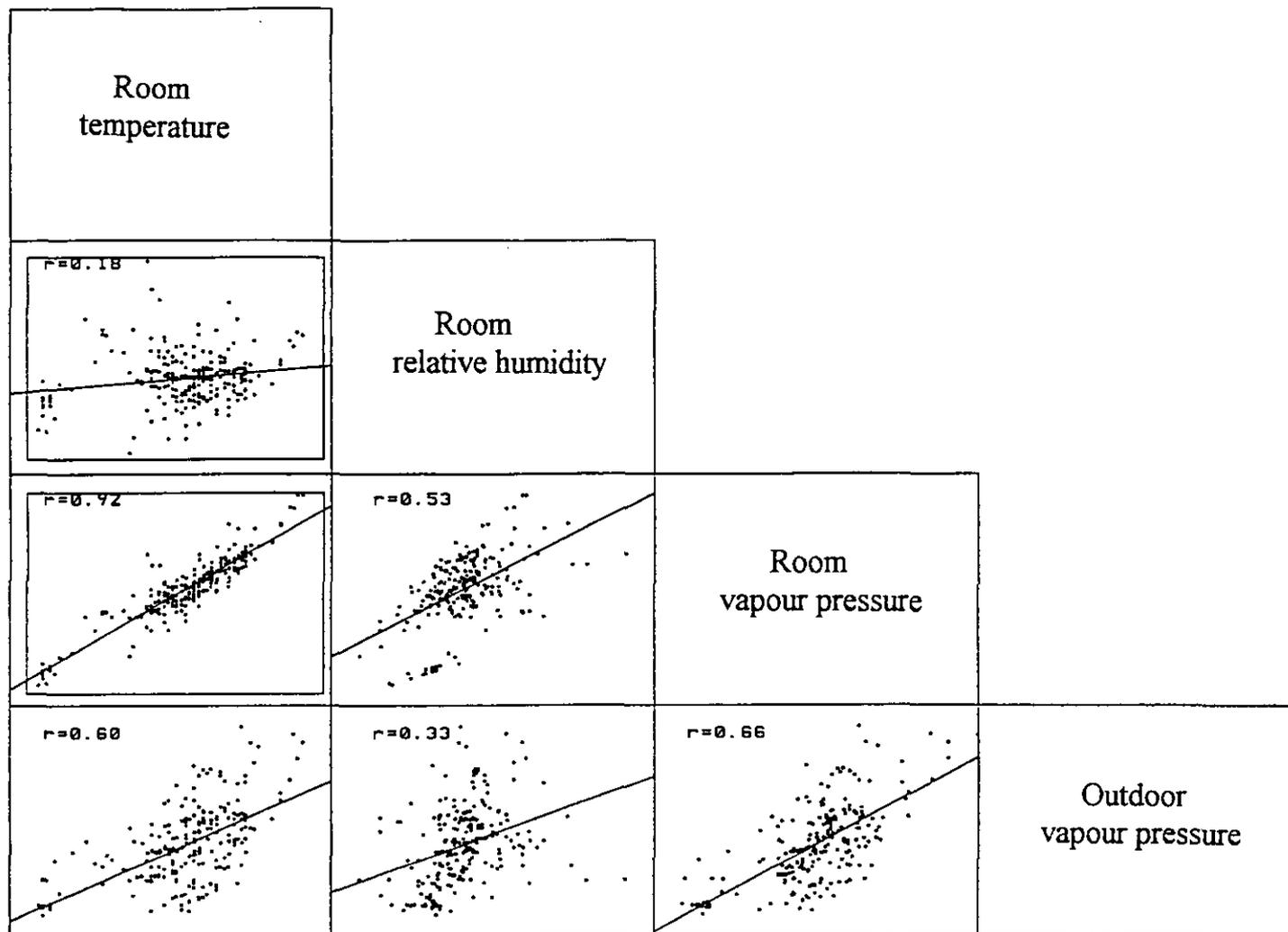


Fig. 3—continued.

where p_s is the saturated vapour pressure in the material, then

$$\Delta p = k_h \Delta m \quad (7)$$

where

$$k_h = l_h p_s \quad (8)$$

Note that since l_h is temperature independent, this expression holds at all temperatures, in particular

$$\bar{k}_h = l_h \bar{p}_s \quad (9)$$

where \bar{k}_h is the value of k_h at the temperature corresponding to \bar{p}_s , the mean saturated vapour pressure in the material.

With this definition of k_h equation (5) becomes

$$V_h \frac{\partial m_h}{\partial t} = A k_h \left(\frac{m_o - m_h}{r} \right) \quad (10)$$

where k_h is the coefficient appropriate in the range m_o to m_h , implying by comparison of equation (4) with (10) that

$$r = k_h R \quad (11)$$

which is the sought-for relationship.

In addition to equation (5), conservation of moisture within the room gives

$$V_o \frac{\partial c_o}{\partial t} = V_o F (c_e - c_o) + \frac{A(p_h - p_o)}{r} + S \quad (12)$$

where S is the moisture source strength (kg s^{-1}) and F is the air-change rate (s^{-1}).

The driving forces for this model are the external vapour pressure and the room temperature taken as sinusoidal. They are given here in complex notation and take the form

$$\begin{aligned} p_e &= \bar{p}_e + \Delta p_e e^{j\omega t} \\ T_r &= \bar{T}_r + \Delta T_r e^{j\omega(t+\theta_r)} \end{aligned} \quad (13)$$

Phase is being referred to outdoor vapour pressure.

If the hygroscopic materials are chiefly room contents then their mean temperature will be the mean value of the room temperature; if on the other hand the hygroscopic materials are chiefly external wall linings then their mean temperature will be slightly modified by the external mean temperature. In the latter case the mean temperature of the hygroscopic materials is given by

$$\bar{T}_h = \frac{h\bar{T}_r + \bar{T}_e/R_{wall}}{h+1/R_{wall}} \quad (14)$$

The sinusoidal amplitude of the temperature fluctuation

tuations of the hygroscopic materials can be taken as equal in magnitude and phase to that of the room temperature. This follows since the effective depth for temperature for non-metallic building materials is tens of centimetres while that for moisture, which is of interest here, is of the order of millimetres. Effectively then the temperature of the thin layer of hygroscopic moisture being transferred in and out of the material is the same as the surface temperature of the material. Further consideration of the relative size of the Biot number shows that this sinusoidal surface temperature is very close in magnitude and phase to that of the room temperature. Specifically, from Carslaw and Jaeger [11] an expression for the surface temperature can be derived as

$$\Delta T_{\text{surface}} = \frac{\Delta T_r \sin(\omega t - \delta)}{\sqrt{(1 + \beta)^2 + \beta}} \quad (15)$$

where

$$\beta = \sqrt{\frac{\omega \rho c k}{2h^2}} \quad \text{and} \quad \delta = \tan^{-1} \frac{\beta}{1 + \beta}. \quad (16)$$

For non-metallic building materials and daily periods β is of the order of 10^{-1} to 10^{-2} which when substituted in equations (15) and (16) confirms that the sinusoidal surface temperature is very close in magnitude and phase to that of the room temperature. Hence

$$\Delta T_r e^{j(\omega t + \theta_r)} \cong \Delta T_h e^{j(\omega t + \theta_r)} \quad (17)$$

where θ_r is phase of the temperature of the hygroscopic materials.

Using equation (14) the mean saturated vapour pressure p_s in the surface layers of the room hygroscopic materials can be obtained, while equation (17) implies that the saturated vapour pressures' fluctuations in both the room and in the thin layer of the hygroscopic materials will also be almost identical, i.e. we can write

$$p_s = \bar{p}_s + \Delta p_s e^{j(\omega t + \theta_s)} \quad (18)$$

where

$$\theta_s = \theta_r = \theta_T. \quad (19)$$

The dependent hygroscopic moisture variable is now changed from moisture content m_h to the corresponding relative humidity ϕ_h . This variable is chosen because, unlike vapour pressure, it is virtually temperature independent, and because the room relative humidity and the hygroscopic material relative humidity tend to equilibrate, again independently of temperature.

We have

$$\frac{\partial m_h}{\partial t} = \frac{\partial m_h}{\partial \phi_h} \frac{\partial \phi_h}{\partial t} = \frac{\bar{p}_s}{k_h} \frac{\partial \phi_h}{\partial t} \quad (20)$$

from equations (6) and (9). This, together with the ideal gas law

$$p = \frac{c RT}{W}$$

where T here is being taken as the mean room temperature T_r (this is equivalent to saying that the mass flow rate (kg s^{-1}) of air through the room is being taken

as constant, or alternatively can be viewed as an approximation of the order of a few percent), causes equation (5) to become

$$V_h \frac{\bar{p}_s}{k_h} \frac{\partial \phi_h}{\partial t} = A \left(\frac{p_o - \phi_h p_s}{r} \right). \quad (21)$$

The response to the sinusoidal driving forces, after decay of transients, will also be sinusoidal ignoring second and higher harmonics (or equivalently assuming that nonlinear effects are not important), i.e.,

$$\phi_h = \bar{\phi}_h + \Delta \phi_h e^{j(\omega t + \theta_h)} \quad (22)$$

and

$$p_o = \bar{p}_o + \Delta p_o e^{j(\omega t + \theta_o)}. \quad (23)$$

Substituting these expressions for the driving forces and responses into equations (12) and (21) gives the following pair of simultaneous algebraic equations

$$\begin{aligned} j\omega \frac{V_o W}{RT} \Delta p_o e^{j(\omega t + \theta_o)} &= F \frac{V_o W}{RT} (\bar{p}_o + \Delta p_o e^{j\omega t} - \bar{p}_o - \Delta p_o e^{j(\omega t + \theta_o)}) \\ &+ \frac{A}{r} ((\bar{\phi}_h + \Delta \phi_h e^{j(\omega t + \theta_h)}) \\ &\times (\bar{p}_s + \Delta p_s e^{j(\omega t + \theta_s)}) - \bar{p}_o - \Delta p_o e^{j(\omega t + \theta_o)}) + S \\ j\omega V_h \frac{\bar{p}_s}{k_h} \Delta \phi_h e^{j(\omega t + \theta_h)} &= \frac{A}{r} (\bar{p}_o + \Delta p_o e^{j(\omega t + \theta_o)} - (\bar{\phi}_h + \Delta \phi_h e^{j(\omega t + \theta_h)}) \\ &\times (\bar{p}_s + \Delta p_s e^{j(\omega t + \theta_s)})) \end{aligned} \quad (24)$$

enabling the steady (after transient) solution of the mass balance equations to be derived.

Equating the parts of these equations not containing time gives the non-sinusoidal steady-state solutions as

$$\begin{aligned} \bar{\phi}_h &= \frac{\bar{p}_o}{\bar{p}_s} + \frac{1}{2} \frac{\Delta \phi_h \Delta p_s}{\bar{p}_s} \cos(\theta_h - \theta_s) \\ &\cong \frac{\bar{p}_o}{\bar{p}_s} = \bar{\phi}_o \end{aligned} \quad (25)$$

where

$$\bar{p}_o = \bar{p}_e + \frac{S}{V_o F} \frac{RT}{W}. \quad (26)$$

The small term

$$\frac{1}{2} \frac{\Delta \phi_h \Delta p_s}{\bar{p}_s} \cos(\theta_h - \theta_s)$$

arises through the constant term implicit in the product $\Delta \phi_h \Delta p_s e^{j(\omega t + \theta_h)} e^{j(\omega t + \theta_s)}$.

Equation (26) is of course the well-known steady-state formula mentioned in the introduction.

Expanding and simplifying equation (24) and ignoring second harmonics (terms containing $e^{j2\omega t}$) and then comparing first harmonic terms, gives in matrix form

$$\begin{bmatrix} -1 & 1 + (F+j\omega) \frac{V_o W_r}{ART} \\ 1 + j\omega \frac{V_h r}{A k_h} & -1 \end{bmatrix} \begin{bmatrix} \Delta\phi_h e^{j\omega t} \\ \frac{\Delta p_o}{p_s} e^{j\omega t} \end{bmatrix} = \begin{bmatrix} \frac{FV_o W_r}{ART} \frac{\Delta p_c}{p_s} + \frac{\overline{\phi}_h \Delta p_s}{p_s} e^{j\omega t} \\ -\frac{\overline{\phi}_h \Delta p_s}{p_s} e^{j\omega t} \end{bmatrix} \quad (27)$$

If two time constants are defined as

$$\begin{aligned} t_1 &= \frac{V_h r}{A k_h} \\ t_2 &= \frac{V_o W_r}{ART} \end{aligned} \quad (28)$$

then equation (27) becomes

$$\begin{bmatrix} -1 & 1 + (F+j\omega)t_2 \\ 1 + j\omega t_1 & -1 \end{bmatrix} \begin{bmatrix} \Delta\phi_h e^{j\omega t} \\ \frac{\Delta p_o}{p_s} e^{j\omega t} \end{bmatrix} = \begin{bmatrix} Ft_2 \frac{\Delta p_c}{p_s} + \frac{\overline{\phi}_h \Delta p_s}{p_s} e^{j\omega t} \\ -\frac{\overline{\phi}_h \Delta p_s}{p_s} e^{j\omega t} \end{bmatrix} \quad (29)$$

t_1 can be thought of as the response time of the hygroscopic materials to a change in the room vapour pressure and is of the order of 10^4 to 10^5 seconds for building materials, while t_2 can be thought of as the response time of the room vapour pressure to a change in the humidity of hygroscopic materials and is of the order of 10^2 to 10^3 seconds.

The solutions to equation (29) are

$$\Delta p_o e^{j\omega t} = \frac{Ft_2 \Delta p_c + j\omega t_1 (Ft_2 \Delta p_c + \overline{\phi}_h \Delta p_s e^{j\omega t})}{Ft_2 - \omega^2 t_1 t_2 + j\omega(t_1 + t_2 + Ft_1 t_2)} \quad (30)$$

$$\Delta\phi_h e^{j\omega t} = \frac{Ft_2 (\Delta p_c - \overline{\phi}_h \Delta p_s e^{j\omega t}) - j\omega t_2 \overline{\phi}_h \Delta p_s e^{j\omega t}}{p_s (Ft_2 - \omega^2 t_1 t_2 + j\omega(t_1 + t_2 + Ft_1 t_2))} \quad (31)$$

To examine these solutions we derive first an expression for the room relative humidity fluctuations.

By definition we have

$$\begin{aligned} \phi_o &= \overline{\phi}_o + \Delta\phi_o e^{j(\omega t + \theta_o)} \\ &= \frac{p_o}{p_s} + \frac{\overline{p}_o + \Delta p_o e^{j(\omega t + \theta_o)}}{\overline{p}_s + \Delta p_s e^{j(\omega t + \theta_s)}} \\ &\cong \overline{\phi}_o + \frac{\Delta p_o}{p_s} e^{j(\omega t + \theta_o)} - \frac{\Delta p_s}{p_s} e^{j(\omega t + \theta_s)} \end{aligned}$$

performing the division and ignoring second and higher harmonics.

Hence the expression sought for the room relative humidity fluctuation takes the form

$$\Delta\phi_o e^{j\omega t} \cong \frac{\Delta p_o}{p_s} e^{j\omega t} - \overline{\phi}_h \frac{\Delta p_s}{p_s} e^{j\omega t} \quad (32)$$

Using this expression, the solutions for room vapour pressure and hygroscopic humidity fluctuations, equations (30) and (31), can be seen to exhibit expected characteristics in the limiting cases: if the air-change rate F is large then

$$\Delta p_o e^{j\omega t} \rightarrow \Delta p_c \quad (33)$$

and

$$\Delta\phi_h e^{j\omega t} \rightarrow \frac{(\Delta p_o e^{j\omega t} - \overline{\phi}_h \Delta p_s e^{j\omega t})}{p_s (1 + j\omega t_1)} = \frac{\Delta\phi_o e^{j\omega t}}{1 + j\omega t_1} \quad (34)$$

i.e., at high air-change rates, the room vapour pressure follows the external vapour pressure (independently of temperature) and the hygroscopic material relative humidity follows the room relative humidity with an amplitude reduction and phase lag determined by the size of t_1 .

Specifically by taking the amplitude and phase of the complex quantities in equation (34) and using equations (22) and (25) we have, in the limit of high air-change rate

$$\phi_h = \overline{\phi}_o + \frac{\Delta\phi_o \sin(\omega t + \theta_o - \delta)}{\sqrt{1 + (\omega t_1)^2}}$$

where

$$\delta = \tan^{-1} \omega t_1.$$

On the other hand, in the case where air-change is very small we have from equation (30)

$$\Delta p_o e^{j\omega t} \cong \overline{\phi}_h \Delta p_s e^{j\omega t}$$

since $t_2/t_1 \sim 10^{-2}$, i.e.,

$$\Delta\phi_o e^{j\omega t} = \frac{\Delta p_o}{p_s} e^{j\omega t} - \overline{\phi}_h \frac{\Delta p_s}{p_s} e^{j\omega t} \cong 0$$

from equation (32).

From equation (31) we have

$$\Delta\phi_h \cong 0.$$

Hence at this limit of a very air-tight room the room relative humidity remains nearly constant and is controlled by the hygroscopic materials' relative humidity which is also nearly constant; as a consequence room vapour pressure is now strongly temperature dependent.

These remarks on the qualitative performance of tight and loose rooms are precisely mirrored by observations in the field, see Figs 1-3.

In order to extract useful quantitative information from the expressions derived above, equations (30) and (31), equation (31) will now be rearranged to solve for the room air-change rate. This yields

$$Ft_2 = \left(\frac{j\omega t_1}{1 + j\omega t_1} \right) \left(\frac{\Delta p_o e^{j\omega t} (1 + t_2/t_1 + j\omega t_2) - \overline{\phi}_h \Delta p_s e^{j\omega t}}{\Delta p_c - \Delta p_o e^{j\omega t}} \right) \quad (35)$$

Since $t_2/t_1 \sim 10^{-2}$ and $\omega t_2 \sim 10^{-1}$ or 10^{-2} , using equation (32) this approximates as

Table 1. Measured and calculated air-change rates for field study houses

House number	2	3	4	7	8
Relative humidity amplitude (%)	5	3	1.5	4.5	5
Room vapour pressure phase (radians)	0.00	0.26	0.21	0.00	0.50
Room vapour pressure amplitude (Pa)	135	110	100	170	125
External vapour pressure amplitude (Pa)	150	160	175	200	175
Mean room saturated vapour pressure (Pa)	1300	1630	1550	1750	2440
Room volume (m ³)	48	31.5	5.4	57.2	26.6
Room area (m ²)	88	63.5	18.6	100	55.8
t_2 (s)	9500	8600	5050	9950	8300
Calculated air-change rate (h ⁻¹)	1.5	0.4	0.2	0.9	1.1
Measured air-change rate (h ⁻¹)	1.5	0.5	Not measured	0.9	1.2

$$Ft_2 \cong \left(\frac{j\omega t_1}{1 + j\omega t_1} \right) \left(\frac{\bar{p}_s \Delta \phi_o e^{j\theta_o}}{\Delta p_e - \Delta p_o e^{j\theta_o}} \right) \quad (36)$$

The expression states that the air-change rate is proportional to the room relative humidity fluctuations, $\Delta \phi_o$, and inversely proportional to the difference between the indoor and outdoor vapour pressure fluctuations, allowing for phase differences. If the room relative humidity fluctuations are near zero, the room is very air-tight, while if the indoor and outdoor vapour pressure fluctuations are nearly equal then the room is very air-loose.

Taking the amplitude of equation (36) gives

$$Ft_2 = \frac{\omega t_1}{\sqrt{1 + (\omega t_1)^2}} \frac{\bar{p}_s \Delta \phi_o}{\sqrt{(\Delta p_e - \Delta p_o \cos \theta_o)^2 + (\Delta p_o \sin \theta_o)^2}} \quad (37)$$

For that case where the air-change rate is large enough that θ_o is approximately zero, this simplifies to

$$Ft_2 = \frac{\omega t_1}{\sqrt{1 + (\omega t_1)^2}} \frac{\bar{p}_s \Delta \phi_o}{\Delta p_e - \Delta p_o} \quad (38)$$

FIELD RESULTS

The field study has been reported in full elsewhere [9]. Briefly, its chief aim was to obtain field data to allow a numerical model SMAHT to be verified. Seven new-built houses in different climate zones in New Zealand were instrumented, while under construction, for timber moisture contents, and temperatures and relative humidities. Logging was carried out over a period of a year, and during that time mean air-change rates over a period of one month into roof cavities and room spaces of some of the houses were measured, using passive gas emitting and absorbing tubes.

Good quality data were obtained for the purposes of the study (model verification) but also suitable data were obtained allowing the effectiveness of the technique developed here to be tested.

Specifically, indoor and outdoor psychrometric conditions were measured once per hour using thermistors and capacitive relative humidity sensors. Temperature accuracy is about $\pm 0.5^\circ\text{C}$, relative humidity accuracy about $\pm 5\%$, and air-change measurement about $\pm 20\%$, see [9] for further details.

Details required to allow the use of formula (37) are contained in Table 1. Details of a fifth house in the

field study appear in this table, with a calculated but unmeasured air-change rate, included because it represents an extreme case of a tight room. t_2 was calculated from its definition, formula (28), in which the effective resistance r was calculated in turn from its definition, formulae (11), (2) and (7). For all houses a plasterboard figure for the diffusion coefficient was taken, viz. $D_m = 1.0 \times 10^{-9} \text{ m}^2 \text{ s}^{-1}$, as was a surface resistance of $1.0 \times 10^9 \text{ N s kg}^{-1}$, and $k_h = 500 \text{ m}^2 \text{ s}^{-2}$ estimated from the average slope of the sorption curve of plasterboard between 30% and 70% RH at 15°C [12], giving an effective depth of 2.6 mm, an effective resistance of $r = 2.3 \times 10^9 \text{ N s kg}^{-1}$ and $t_1 = 2.4 \times 10^4 \text{ s}$.

Agreement between measured and calculated values is good but dependent upon the accuracy in estimation of t_2 ; it should however be remembered that even with an inaccurate value for t_2 the method should retain usefulness, as it will still give correct relative results.

Indeed, estimating t_2 , in other words estimating the amount and effectiveness of the hygroscopic material in the room, is the greatest difficulty with this scheme. Two approaches suggest themselves:

1. Calibration. A single air-change measurement could be taken with some appropriate technique and this known value used in formula (28) to calculate t_2 . From there onwards, air-change rates could be followed continuously using cheap and easy to operate equipment, the data being substituted into formula (37) with the now known value of t_2 , as changing conditions altered the size of the relative humidity and vapour pressure fluctuations.
2. Direct measurements. Major hygroscopic components in the room could be identified, their area exposed to the room air calculated, and t_2 calculated from the known properties of the materials, i.e. the estimates made in the calculations carried out in this work could be refined.

Estimating the amplitude of the sinusoidal fluctuations in vapour pressures and relative humidity is not always easy, especially over those periods where the driving forces and responses are only approximately sinusoidal. These estimates could be up to 20% inaccurate, with corresponding inaccuracies in the calculation of the air-change rate. The overall inaccuracies become very large if the indoor and outdoor vapour pressures are nearly equal, i.e., if air-change rates are high. In practice, this may not be of major concern—if air-change rates are

high, any error in calculating the effect of these high air-change rates on indoor climate and other factors of interest is correspondingly very small—simply, the indoor vapour pressure will be equal to the outdoor vapour pressure at all air-change rates that are so high that their exact value is difficult to calculate accurately.

Formula (35) derived above holds for any frequency. Short-term fluctuations in internal moisture load caused by activities such as cooking, showering etc., or short-term fluctuations in internal temperature caused by heating etc. can be thought of as parts of high frequency sinusoidal oscillations, with periods determined by the rise time of these fluctuations. The case of high frequency temperature fluctuations can be dealt with in this formula by using the appropriate value of Δp , corresponding to these temperature fluctuations, and by taking $\Delta p_e = 0$ over this short time period. As a result it will be found that at high frequencies the fluctuations in the room vapour pressure due to temperature fluctuations would be zero but for a small contribution from the hygroscopic materials. The formulae derived are not intended to describe the case of short-term fluctuations in internal moisture load; it is found in practice that these loads are simply superimposed upon the approximate daily cycle and do not interfere with estimating the amplitude of the daily fluctuations.

CONCLUSIONS

Data obtained in a field study suggested that ventilation rates were proportional to indoor relative humidity fluctuations, due to hygroscopic damping.

This hygroscopic damping has been quantified, enabling a simple formula to be derived which yields room ventilation rates given the amplitude of sinusoidal fluctuations in external vapour pressure, and fluctuations in room vapour pressure and humidity. It has been found that the room air-change rate is proportional to the room relative humidity fluctuations, and inversely proportional to the difference between the indoor and outdoor vapour pressure fluctuations, allowing for phase differences.

Calculated results using this formula agree well with field data.

The biggest uncertainty with this technique is the need to estimate the quantity and effectiveness of the room hygroscopic materials that result in the damping observed. This suggests that a calibration air-change measurement should first be undertaken to quantify the influence of the room hygroscopic materials (through a time constant t_2) which can be used thereafter in the formula derived. A technique would then be available which would give approximate but continuous values of air-change using simple and inexpensive equipment.

REFERENCES

1. R. Dales, H. Zwanenburg and R. Burnett, The Canadian air quality health survey: influence of home dampness and molds on respiratory health, *Proceedings of the 5th International Conference on Indoor Air Quality, Indoor Air '90*, Toronto, July 29–Aug 3, 1990, 1, p. 145 (1990).
2. P. Fairy and A. Kerestecioglu, Dynamic modelling of combined thermal and moisture transport in buildings: effects on cooling loads and space conditions. *ASHRAE Trans.* 91, part 2A, 461–472 (1985).
3. C. Isetti, L. Laurenti and A. Ponticciello, Predicting vapour content of the indoor air and latent loads for air-conditioning environments: effect of moisture storage capacity of the walls, *Energy and Buildings*, 12, 141–148 (1988).
4. A. A. Kerestecioglu, M. V. Swami and A. A. Kamel, Theoretical and computational investigation of simultaneous heat and moisture transfer in buildings: "effective penetration depth" theory, *ASHRAE Trans* 96, part 1, 447–454 (1990).
5. R. El Diasty, P. Fazio and I. Budaiwi, The dynamic modelling of air humidity behaviour in a multi-zone space, *Bldg Envir.*, 28, 33–51 (1993).
6. P. Martin and J. Verschoor, Cyclical moisture desorption/absorption by building construction and furnishing materials, Symposium on Air Infiltration, Ventilation and Moisture Transfer, Building Thermal Envelope Coordinating Council, Fort Worth, Texas, 59–69 (1986).
7. R. El Diasty, P. Fazio and I. Budaiwi, Dynamic modelling of moisture absorption and desorption in buildings, *Bldg Envir.*, 28, 21–32 (1993).
8. M. J. Cunningham, Effective penetration depth and effective resistance in moisture transfer, *Bldg Envir.*, 27, 379–386 (1992).
9. M. J. Cunningham, M. R. Bassett, D. McQuade and M. Beckett, A field study of the moisture performance of roofs of occupied newly constructed timber framed houses. *Bldg Envir.* 29, 173–190 (1994).
10. M. R. Bassett and H. M. Beckett, Automated tracer equipment for air flow studies in buildings, *Proceedings of 10th AIVC Conference*, Espoo, Finland, 25–28 Sept, 1989, 2, 293 (1989).
11. H. S. Carslaw and J. C. Jaeger, *Conduction of Heat in Solids*. Oxford University Press, London (1959).
12. M. J. Cunningham and T. J. Sprott, Sorption properties of New Zealand building materials. Research report R43, Building Research Association of New Zealand, Judgeford (1984).



BRANZ MISSION

To be the leading resource for the development of the building and construction industry.

HEAD OFFICE AND RESEARCH CENTRE

Moonshine Road, Judgeford
Postal Address - Private Bag 50908, Porirua
Telephone - (04) 235-7600, FAX - (04) 235-6070

REGIONAL ADVISORY OFFICES

AUCKLAND

Telephone - (09) 524-7018
FAX - (09) 524-7069
118 Carlton Gore Road, Newmarket
PO Box 99-186, Newmarket

WELLINGTON

Telephone - (04) 235-7600
FAX - (04) 235-6070
Moonshine Road, Judgeford

CHRISTCHURCH

Telephone - (03) 366-3435
FAX - (03) 366-8552
GRE Building
79-83 Hereford Street
PO Box 496

Using hygroscopic damping of relative humidity
pressure fluctuations to measure room ventilation
CUNNINGHAM, M.J.

Dec 1994
32789

Copy 2



BRANZ MISSION

To be the leading resource for the development of the building and construction industry.

HEAD OFFICE AND RESEARCH CENTRE

Moonshine Road, Judgeford
Postal Address - Private Bag 50908, Porirua
Telephone - (04) 235-7600, FAX - (04) 235-6070

REGIONAL ADVISORY OFFICES

AUCKLAND

Telephone - (09) 524-7018
FAX - (09) 524-7069
118 Carlton Gore Road, Newmarket
PO Box 99-186, Newmarket

WELLINGTON

Telephone - (04) 235-7600
FAX - (04) 235-6070
Moonshine Road, Judgeford

CHRISTCHURCH

Telephone - (03) 366-3435
FAX - (03) 366-8552
GRE Building
79-83 Hereford Street
PO Box 496